A1

... carry equal marks.

Q1 (a) Solve $(D-1)^2(D+1)^2 = Sin^2\left(\frac{x}{2}\right)+e^x+x$

(b) Express the matrix $\begin{bmatrix} 2 & 1 & 3 \\ -1 & 4 & 1 \\ 0 & 2 & -2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Q2 (a) Show that the values of $\left(1+i\sqrt{3}\right)^{1/3}$ are given by $2^{\frac{1}{3}}\left(\cos\frac{r\pi}{9}+i\sin\frac{r\pi}{9}\right)$, $r=1,7,13,...$

(b) If $f(x)=x^2e^{-x/a}$ , show that $f_n(0)=\frac{n(n-1)(-1)^n}{a^{n-2}}$ where $f_n=\frac{d^n f}{dx^n}$

Q3 (a) The tangents at two points P and Q on the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ are at right angles. If $\rho_1=\rho_2$ are the radii of curvature at these points then show that $\rho_1^2+\rho_2^2=16a^2$.

(b) Solve $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x^4\sin x$.

A2

Q4 (a) Show that $\int_0^a \frac{x^n dx}{\sqrt{ax-x^2}}=\frac{1.3.5..(2n-1)}{2.4.6...2n}\pi a^n$

(b) Fine the area outside the circle r=2acosθ and inside the cardioid r=a(1+cosθ).

Q5 (a) Reduce the following matrix into its normal form and hence find its rank

$$A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

(b) Trace the curve $y^2(2a-x)=x^3$.

Q6 (a) Show that the asymptotes of the cubic curve $x^3+x^2y-xy^2-y^3-2x^2+2y^2-5x+y+1=0$ cut the curve in three points which lie on the straight line $2x+2y=1$.

(b) Test for convergence, the series

$$\frac{1}{3}+\frac{3}{5}+\frac{7}{9}+\frac{15}{17}+...$$

Q7 (a) Show that the system of equations

$$x+y+3z=6$$
$$2x+3y+z=8$$
$$x+5y+7z=20$$
$$x+z=10$$

has no solution.

(b) If log $\log\sin(\theta+i\phi)=x+iy$, show that $2\cos 2\theta=e^{2\phi}+e^{-2\phi}-4e^{2x}$

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P

A3

...Attempt any five questions...
*Select one question from each Unit.*

Q1 (a) Find the $n^{th}$ derivative of $\sin 2x \sin 3x$. **(3)**

(b) Find all the asymptotes of the curve $y^2(x-2a) = x^3 - a^3$. **(3)**

(c) Find the value of $\int_0^{\pi/2} \sin^3\theta \cos^{5/2}\theta d\theta$. **(3)**

(d) Prove that every Hermitian matrix can be written as $A+iB$, where A is real and symmetric and $B$ is real and skew symmetric matrix. **(3)**

(e) Reduce the quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ to the canonical form. Also write the nature of the quadratic form. **(3)**

(f) Solve the differential equation $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$. **(3)**

(g) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(3)**

(h) Prove that $\int_{-1}^{1} P_n(x)dx = 0 \qquad n \neq 0$. **(2)**

(i) Discuss the convergence of the series, $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots\dots\infty$, using Leibnitz's rule. **(2)**

**Unit-I**

Q2 (a) If $y = e^{m(\cos^{-1}x)}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. Also find $(y_n)_0$. **(6.5)**

(b) Discuss the convergence of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \dots\dots\infty$. **(6)**

A4

Q3 (a) Show that $\tan\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 + \frac{8}{3}x^3 + \frac{10}{3}x^4 + \dots\dots$ and hence calculate the value of $46^0$ correct up to 4 places of decimal. **(6.5)**

(b) Discuss the absolute convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)}$. **(6)**

**Unit-II**

Q4 (a) Show that the radius of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $a/\sqrt{2}$.

(b) Trace the curve $r = a\cos 3\theta$.

**(6)**

**(6.5)**

Q5 (a) Find the area include between the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and its base.

(b) Evaluate $\int \sin^6 x$ using reduction formula.

**(6.5)**

**(6)**