TRUSS

Truss is a structure that is made up of straight slender bars that are joined together at their ends by frictionless pins to form a pattern of triangle. Every member of a truss is a two force member.

PERFECT, DEFICIENT AND REDUNDANT TRUSSES

A truss which has got just sufficient number of members to resist the loads without undergoing deformation in its shape is called a perfect truss.
Triangular truss is the simplest perfect truss and it has three joints and three members.

For a perfect truss m = 2j – 3
Where, total number of members = m   total number of joints = j

In a deficient truss the number of members in it are less than that required for a perfect truss. Such trusses cannot retain their shape when loaded.
In a redundant truss the number of members in it are more than that required in a perfect truss. Such trusses cannot be analysed by making use of the equations of equilibrium alone. Thus, a redundant truss is statically indeterminate.

ASSUMPTIONS FOR FORCES IN THE MEMBERS OF A PERFECT TRUSS

Following assumptions are made, while finding out the forces in the members of a perfect truss:
1. All the members are pin-jointed (hinged).
2. The loads act only at the joints.
3. Self-weights of the members are negligible.
4. Cross-section of the members is uniform.

METHODS OF ANALYSIS FOR THE FORCES IN THE MEMBERS OF A TRUSS

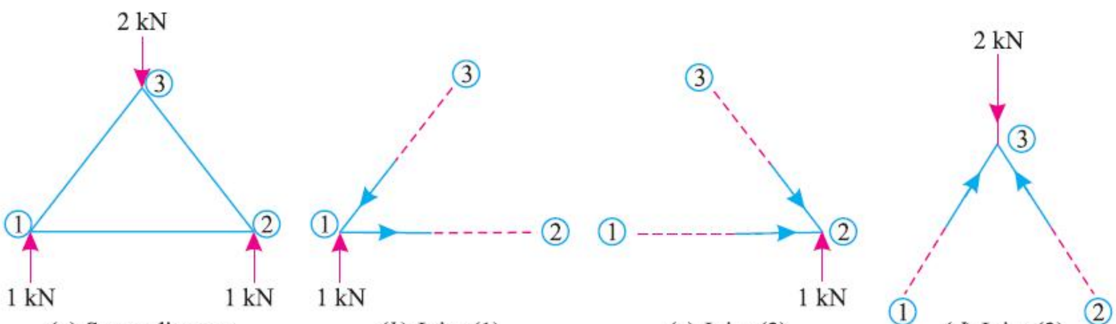
The following two analytical methods are used to finding out the forces, in the members of a perfect truss:

1. Method of joints. 2. Method of sections

METHOD OF JOINTS

In this method, each and every joint is treated as a free body in equilibrium as shown in Fig.
The unknown forces are then determined by using equilibrium equations i.e. $\sum V = 0$ and $\sum H = 0$. *i.e.,* Sum of all the vertical forces and horizontal forces is equated to zero.

Notes: While selecting the joint, for calculation work, care should be taken that at any instant, the joint should not contain more than two members, in which the forces are unknown.



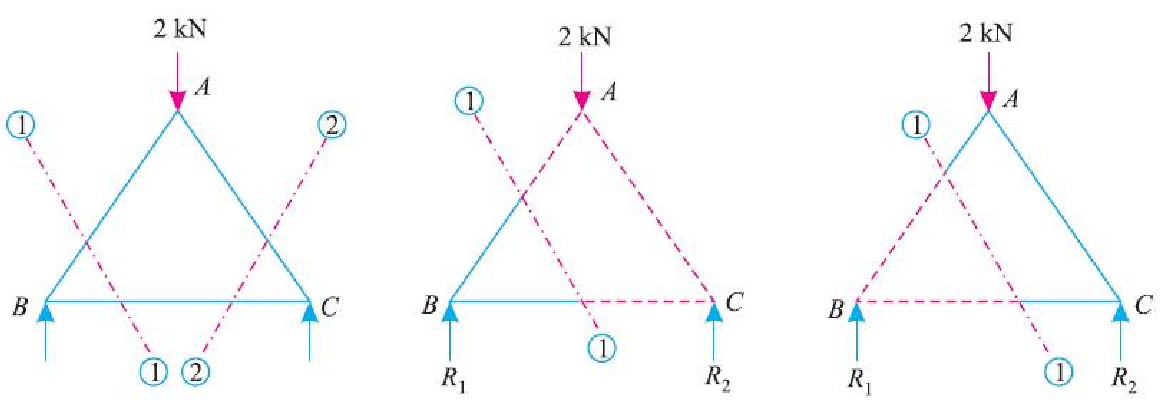
(*a*) Space diagram (*b*) Joint (1) (*c*) Joint (2) (*d*) Joint (3)

METHOD OF SECTIONS

This method is particularly convenient, when the forces in a few members of a frame are required to be found out. In this method, a section line is passed through the member or members, in
which the forces are required to be found out.

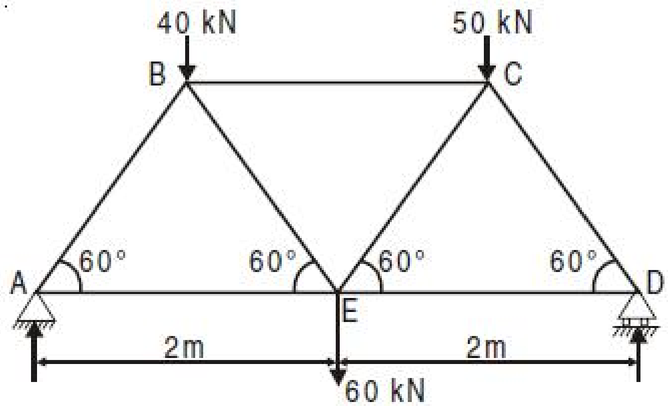
The unknown forces are then found out by the application of equilibrium equations:
$\sum V = 0$, $\sum H = 0$ *and* $\sum M = 0$.

Notes:1. To start with, we have shown section line 1-1 cutting the members *AB* and *BC*. Now in order to find out the forces in the member *AC*, section line 2-2 may be drawn.
2. While drawing a section line, care should always be taken not to cut more than three members, in which the forces are unknown.



(*a*) Space diagram (*b*) Left part (*c*) Right part

Example: *Determine the forces in all the members of the truss shown below and indicate the magnitude and nature of forces on the diagram of the truss. All inclined members are at 60° to horizontal and length of each member is 2 m.*



Solution: Now, we cannot find a joint with only two unknown forces without finding reactions. Consider the equilibrium of the entire frame.

$\sum MA = 0$, gives

$RD \times 4 - 40 \times 1 - 60 \times 2 - 50 \times 3 = 0$

$RD = 77.5$ kN

$\sum H = 0$, gives

$HA = 0$

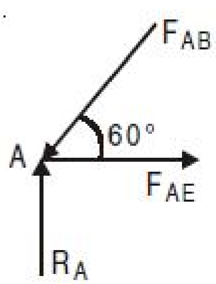
Reaction at *A* is vertical only

$\sum V = 0$, gives

$RA + 77.5 = 40 + 60 + 50$

$RA = 72.5$ kN

Now consider FBD of Joint *A*:



$\sum V = 0$, gives

$FAB \sin 60° = RA = 72.5$

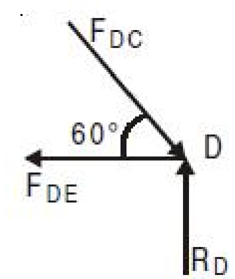
***FAB* = 83.7158 kN (Comp.)**

$\sum H = 0$, gives

FAE – 83.7158 cos 60° = 0

***FAE* = 41.8579 kN (Tension)**

Now consider FBD of Joint *D*:



$\sum V$ = 0, gives

FDC sin 60° = *RD* = 77.5

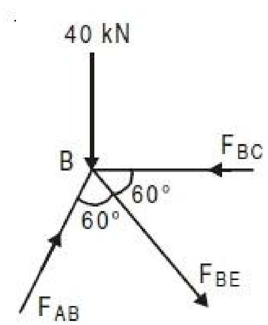
***FDC* = 89.4893 kN (Comp.)**

$\sum H$ = 0, gives

FDE – 87.4893 cos 60° = 0

***FDE* = 44.7446 kN (Tension)**

Consider FBD of Joint *B:*



$\sum V$ = 0, gives

FBE sin 60° – *FAB* sin 60° + 40 = 0

FBE = 37 5278 (Tension)

$\sum H$ = 0, gives

FBC – *FAB* cos 60° – *FBE* cos 60° = 0

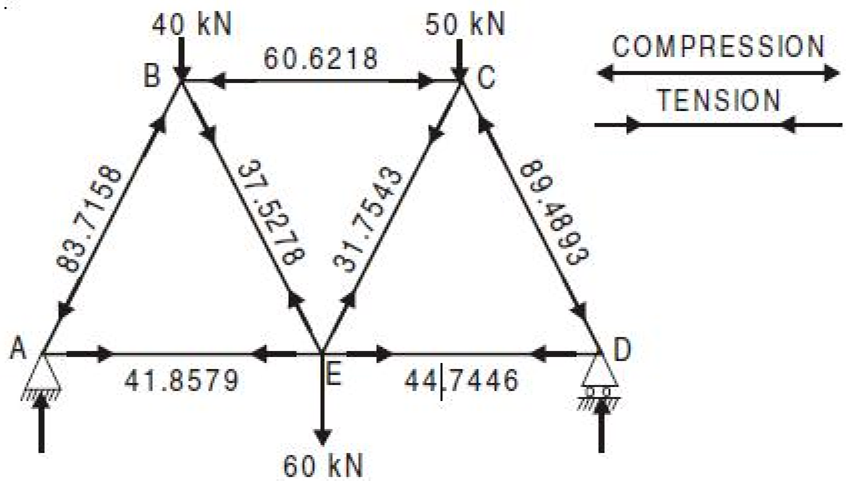
FBC = (83.7158 + 37.5274) × 0.5

***FBC* = 60.6218 kN (Comp.)**

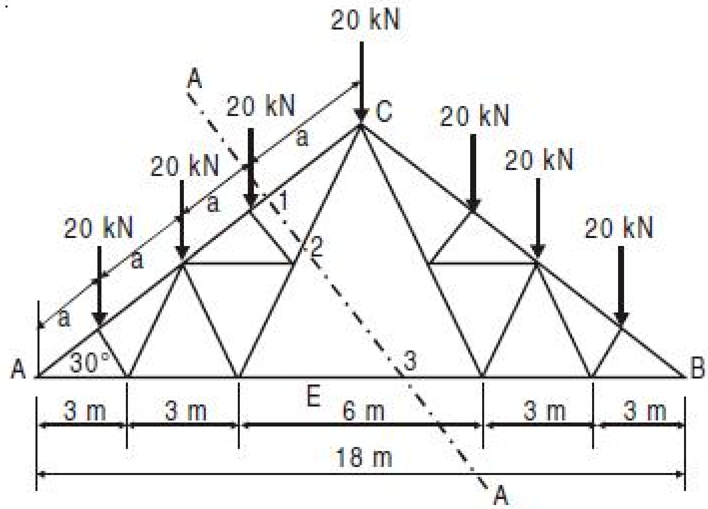
Similarly Joint *C* gives:

FCE sin 60° + 50 – *FDC* sin 60° = 0

***FCE* = 31.7543 kN (Tension)**


40 kN
50 kN
60.6218
B
C
COMPRESSION
TENSION
83.7158
37.5278
31.7543
89.4893
A
D
E
41.8579
44.7446
60 kN

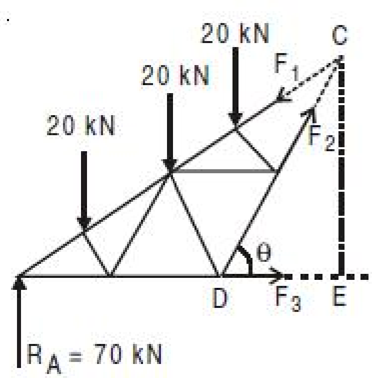
Example : *Find the forces in the members (1), (2) and (3) of truss shown below.*



***Solution*:** Due to symmetry

$RA = RB = 70$ kN

Now, $AC = 4 \times a = 9/\cos 30°$
$a = 2.5981$ m.



Take Section (*A*)–(*A*) and consider the equilibrium of left hand side part of the Truss.

Drop perpendicular *CE* on *AB.*

Now, $CE = 9 \tan 30°$ and $DE = 3$ m

$\theta = 60°$
$\sum MA = 0$, gives

$F2 \sin 60° \times 6 - 20 \times 2.5981 \cos 30° - 20 \times 2 \times 2.5981 \cos 30° - 20 \times 3 \times 2.5981 \cos 30° = 0$

$F2 = 51.9615$ kN (Tension)

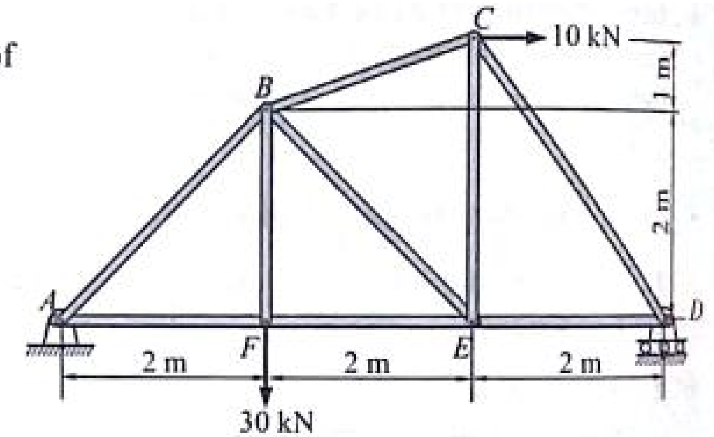
$\sum V = 0$, gives
$F1 \sin 30° - 70 + 20 + 20 + 20 - 51.9615 \sin 60° = 0$
$F1 = 110$ kN (Comp.)

$\sum H = 0$, gives
$F3 + F2 \cos 60° - F1 \cos 30° = 0$
$F3 = 69.2820$ (Tension)

PRACTICE QUESTIONS

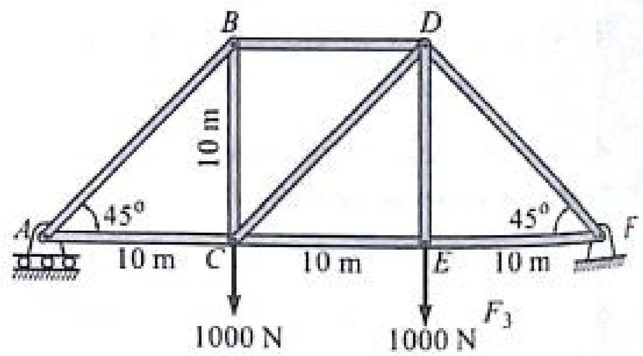
Determine the forces in the members of the truss as shown in Fig. 7.E2.

Ans.
F_{AB} = 21.21 kN (C), F_{BC} = 0,
F_{CD} = 18 kN (C), F_{BE} = 21.21 kN (C),
F_{AF} = 25 kN (T), F_{DE} = 10 kN (T),
F_{CE} = 15 kN (T), F_{EF} = 25 kN (T) and
F_{CE} = 30 kN (T).



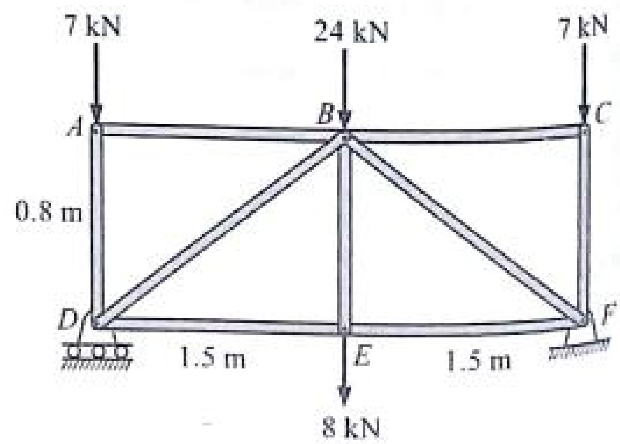
A simple plane truss is shown in Fig. 7.E3. Two 1000 N loads are shown acting on pins C and E. Determine the force in all the members using method of joints.

Ans.
F_{AB} = 1414 N (C), F_{CE} = 1000 N (T),
F_{AC} = 1000 N (T), F_{DE} = 1000 N (T),
F_{BC} = 1000 N (T), F_{EF} = 1000 N (T),
F_{BD} = 1000 N (C), F_{DF} = 1414 N (C) and
F_{CD} = 0.



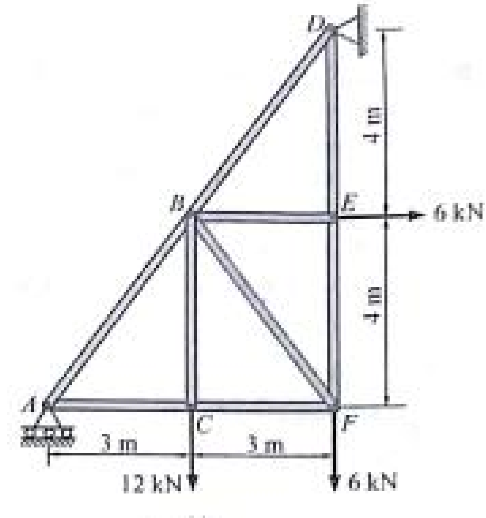
Find the forces in all the members of the truss shown in Fig. 7.E4.

Ans.
$V_D = V_F$ = 23 kN (↑), H_F = 0,
$F_{AD} = F_{CF}$ = 7 kN (C),
$F_{BD} = F_{BF}$ = 34 kN (C),
$F_{DF} = F_{EF}$ = 30 kN (T),
F_{BE} = 8 kN (T) and $F_{AB} = F_{BC}$ = 0.



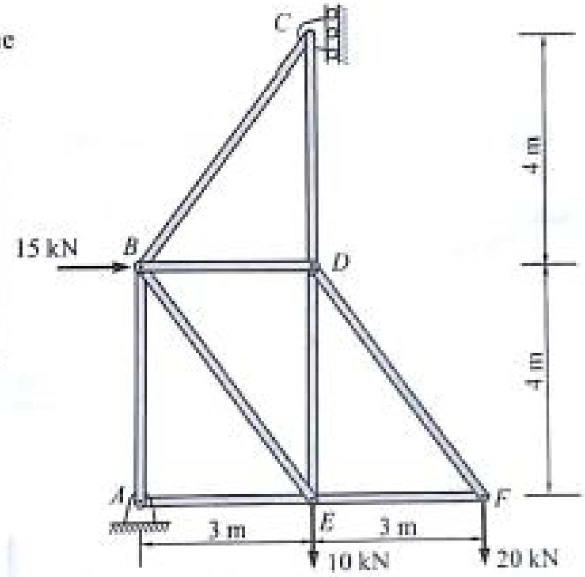
Find the forces in all the members of the truss loaded as shown in Fig. 7.E5. Also, determine the force in the member BF by method of section.

Ans.
F_{AB} = 12.5 kN (C), F_{EF} = 16 kN (T),
F_{AC} = 7.5 kN (T), F_{BE} = 6 kN (T),
F_{BC} = 12 kN (T), F_{DE} = 16 kN (T),
F_{CF} = 7.5 kN (T), F_{BD} = 10 kN (C) and
F_{BF} = 12.5 kN (C).



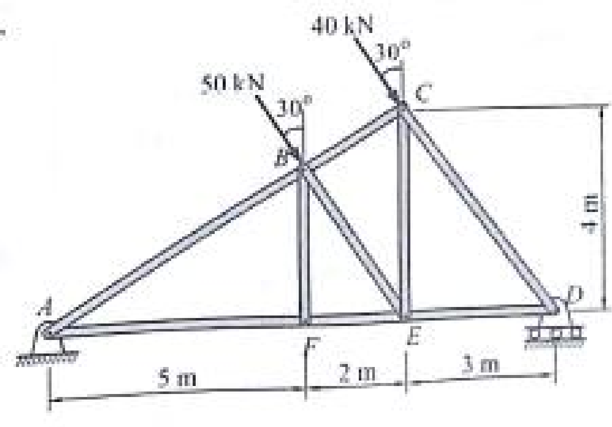
Find the forces in all the members of the truss shown in Fig. 7.E6.

Ans.
F_{CD} = 35 kN (T), F_{DF} = 25 kN (T),
F_{DE} = 15 kN (T), F_{AB} = 30 kN (C),
F_{BC} = 43.75 kN (C),
F_{EF} = 15 kN (C), F_{AE} = 11.25 kN (C),
F_{BE} = 6.2 kN (C) and F_{BD} = 15 kN (C).



Determine the forces in the members of the truss as shown in Fig. 7.E7.

Ans.
F_{CD} = 76.3 kN (C), F_{BC} = 29.89 kN (C),
F_{AB} = 34.07 kN (C), F_{BE} = 50.21 kN (C),
F_{CE} = 41.13 kN (T), F_{AF} = 74.58 kN (T),
F_{EF} = 74.58 kN (T), F_{DE} = 45.78 kN (T)
and F_{BF} = 0.



REFERENCES

A TEXTBOOK OF ENGINEERING MECHANICS by R.S. KHURMI
MECHANICS OF SOLIDS by S. S. BHAVIKATTI
ENGINEERING MECHANICS by N H Dubey

CENTRE OF GRAVITY

Centre of gravity can be defined as the point through which the resultant of force of gravity of the body acts**.**

CENTROID

The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass.
The centre of area of such figures is known as centroid. The method of finding out the centroid of a figure is the same as that of finding out the centre of gravity of a body.

CENTER OF MASS

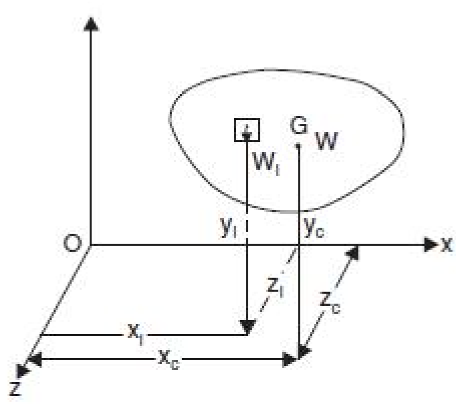
The point of an object at which all the mass of the object is thought to be concentrated.

METHODS FOR CENTRE OF GRAVITY

The centre of gravity (or centroid) may be found out by any one of the following methods:
1. By geometrical considerations
2. By moments
3. By graphical method

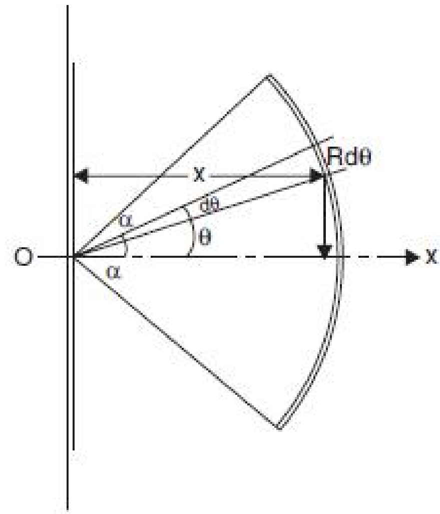
CENTRE OF GRAVITY OF A BODY

Let Wi be the weight of an element in the given body.
W be the total weight of the body.
Let the coordinates of the element be xi, yi, zi and that of centroid G be xc, yc, zc.
Since W is the resultant of Wi forces, Then
W = W1 + W2 + W3 + . . .
$=\sum Wi$



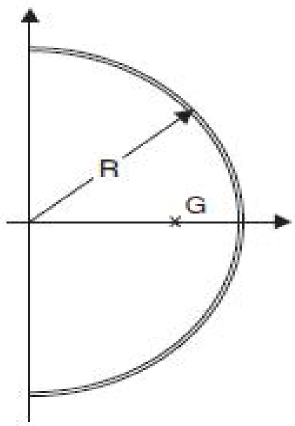
CENTROID OF AN ARC OF A CIRCLE

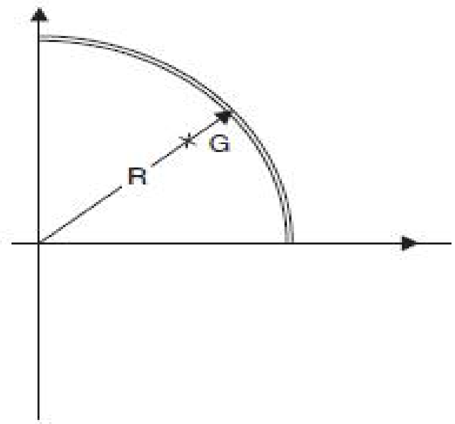
$$L = \text{Length of arc} = R\,2\alpha$$
$$dL = Rd\theta$$



$$x_c L = \int_{-\alpha}^{\alpha} x dL$$
$$x_c\, R\, 2\alpha = \int_{-\alpha}^{\alpha} R\cos\theta \cdot Rd\theta$$
$$= R^2 \Big[\sin\theta\Big]_{-\alpha}^{\alpha}$$
$$x_c = \frac{R^2 \times 2\sin\alpha}{2R\alpha} = \frac{R\sin\alpha}{\alpha}$$
$$y_c\, L \int_{-\alpha}^{\alpha} y\, dL = \int_{-\alpha}^{\alpha} R\sin\theta \cdot Rd\theta$$
$$= R^2 \Big[\cos\theta\Big]_{-\alpha}^{\alpha}$$
$$= 0$$
$$y_c = 0$$

CENTROID OF A SEMICIRCLE AND QUARTER CIRCLE





For semicircle $$x_c = \frac{2R}{\pi}$$

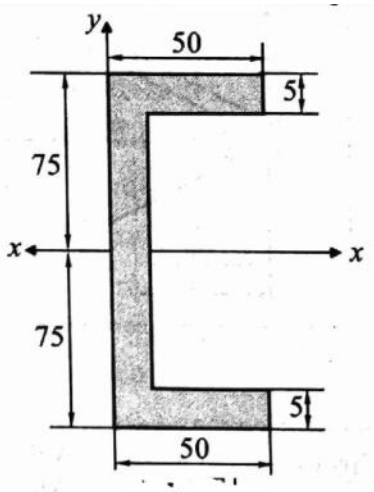
$$y_c = 0$$

For quarter of a circle,

$$x_c = \frac{2R}{\pi}$$

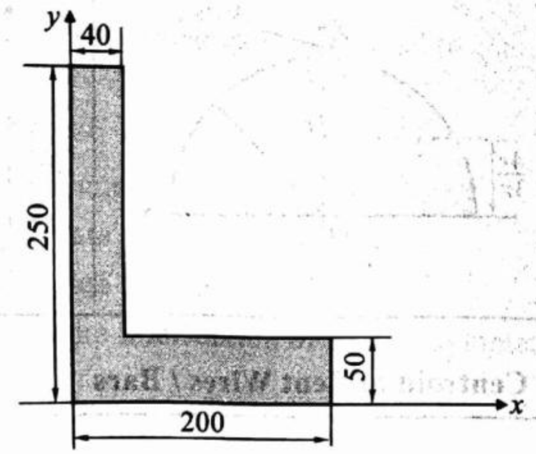
$$y_c = \frac{2R}{\pi}$$

PRATICE QUESTIONS

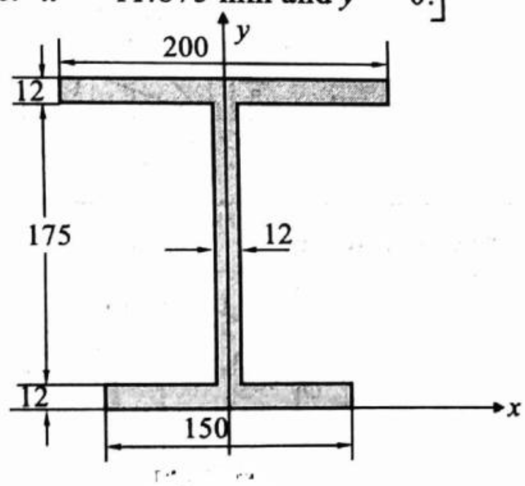


[*Ans.* $\overline{x}$ = 11.875 mm and $\overline{y}$ = 0.]

2.

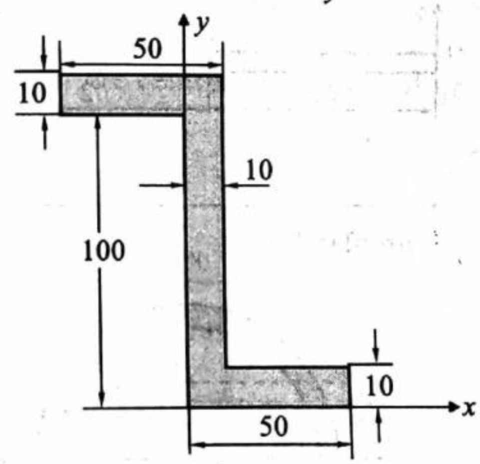


[*Ans.* $\overline{x}$ = 64.4 mm and $\overline{y}$ = 80.5 mm.]



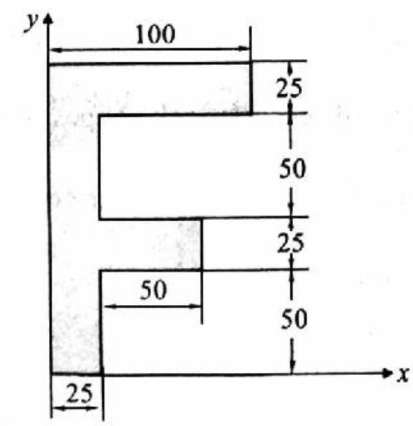
[*Ans.* $\overline{x}$ = 0 and $\overline{y}$ = 108.4 mm.]

4.



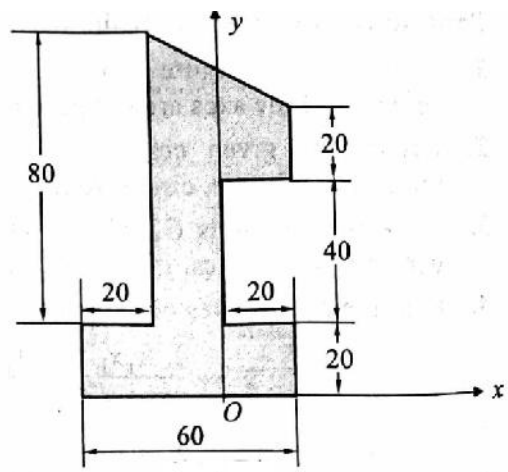
[*Ans.* $\overline{x}$ = 5 mm and $\overline{y}$ = 55 mm.]

5.



[Ans. $\bar{x}$ = 32.95 mm and $\bar{y}$ = 89.77 mm.]

6.



[Ans. $\bar{x}$ = −7.1 mm and $\bar{y}$ = 42.1 mm.]

REFERENCES

A TEXTBOOK OF ENGINEERING MECHANICS by R.S. KHURMI
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MOMENT OF INERTIA

Moment of inertia gives a quantitative estimate of the relative distribution of area and mass of a body with respect to some reference axis is termed as moment of inertia of a body.

MOMENT OF INERTIA OF A PLANE AREA

Consider a plane area, whose moment of inertia is required to be found out. Split up the whole area into a number of small elements.
Let a1, a2, a3, ... = Areas of small elements, and
r1, r2, r3, ... = Corresponding distances of the elements from the line about which the moment of inertia is required to be found out.
Now the moment of inertia of the area is

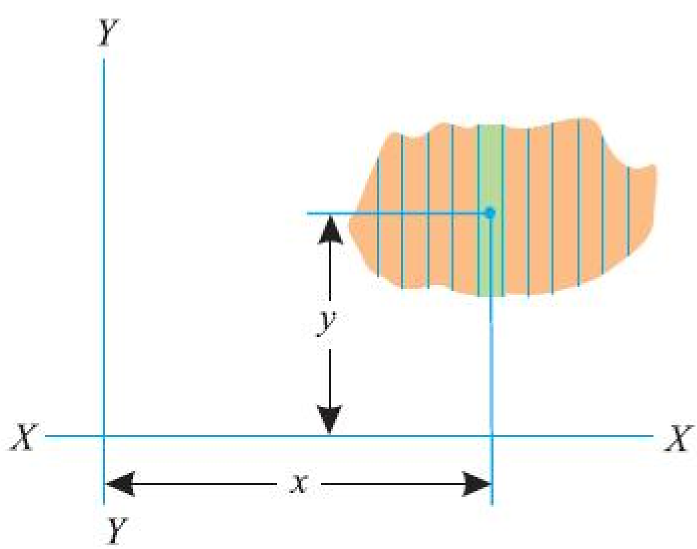
$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + ...$
$= \Sigma a r^2$

UNITS OF MOMENT OF INERTIA

As a matter of fact the units of moment of inertia of a plane area depend upon the units of the area and the length. e.g.,
1. If area is in m^2 and the length is also in m, the moment of inertia is expressed in m^4.
2. If area in mm^2 and the length is also in mm, then moment of inertia is expressed in mm^4.

MOMENT OF INERTIA BY INTEGRATION

The moment of inertia of an area may also be found out by the method of integration as discussed below:
Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and

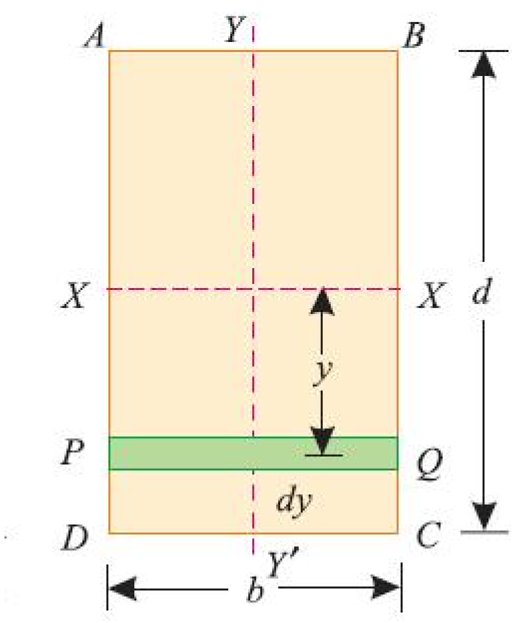


Y-Y axis as shown in figure.

Let us divide the whole area into a no. of strips. Consider one of these strips.
Let dA = Area of the strip
x = Distance of the centre of gravity of the strip on X-X axis and
y = Distance of the centre of gravity of the strip on Y-Y axis.
We know that the moment of inertia of the strip about Y-Y axis
= dA .
Now the moment of inertia of the whole area may be found out by integrating above equation.
i.e.,
$I_{YY} = \Sigma\, dA \,.\, x^2$.
Similarly $I_{XX} = \Sigma\, dA \,.\, y^2$.
In the following pages, we shall discuss the applications of this method for finding out the moment of inertia of various cross-sections.

MOMENT OF INERTIA OF A RECTANGULAR SECTION

Consider a rectangular section ABCD as shown in figure whose moment of inertia is required to be found out.
Let b = Width of the section and
d = Depth of the section.
Now consider a strip PQ of thickness dy parallel to X-X axis and at a distance y from it as shown in the figure



∴ Area of the strip = b.dy
We know that moment of inertia of the strip about X-X axis,
= Area × y^2 = (b. dy) y^2 = b. y^2. dy

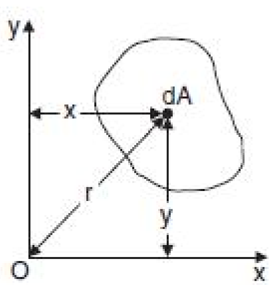
$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy = b \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \, .dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right] = \frac{bd^3}{12}$$

Similarly, $$I_{YY} = \frac{db^3}{12}$$

POLAR MOMENT OF INERTIA

Moment of inertia about an axis perpendicular to the plane of an area is known as polar moment of inertia. It may be denoted as J or Izz.
Thus, the moment of inertia about an axis perpendicular to the plane of the area at O is called polar moment of inertia at point O, and is given by
$I_{zz} = \sum r^2 \; dA$



RADIUS OF GYRATION

Radius of gyration of a body about an axis of rotation is defined as the radial distance of a point from the axis of rotation at which, if whole mass of the body is assumed to be concentrated, its moment of inertia about the given axis would be the same as with its actual distribution of mass.

$$I = mk^2 \quad or \quad k = \sqrt{\frac{I}{m}}$$

PARALLEL AXIS THEOREM

The moment of inertia about any axis parallel to and at distance d away from the axis that passes through the centre of mass is:

$$I_O = I_G + md^2$$

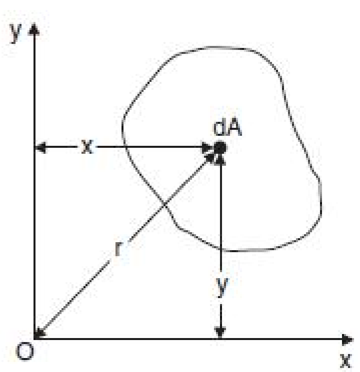
Where

- I_G= moment of inertia for mass centre G
- m = mass of the body

d = perpendicular distance between the parallel axes

PERPENDICULAR AXIS THEOREM

The moment of inertia of an area about an axis pependicular to its plane (polar moment of inertia) at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point O and lying in the plane of the area.
Referring to Figure, if z-z is the axis normal to the plane of paper passing through point O, as per this theorem,
Izz = Ixx + Iyy
The above theorem can be easily proved. Let us consider an elemental area dA at a distance r from O. Let the coordinates of dA be x and y.
Then from definition:
Izz = $\sum r^2$ dA
= $\sum (x^2 + y^2)$dA
= $\sum x^2$dA + $\sum y^2$ dA
Izz = Ixx + Iyy



MOMENT OF INERTIA OF COMPOSITE BODIES

1. Divide the composite area into simple body.

2. Compute the moment of inertia of each simple body about its centroidal axis from table.

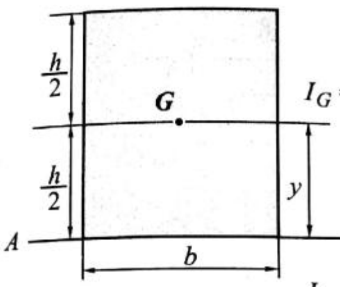
3. Transfer each centroidal moment of inertia to a parallel reference axis

4. The sum of the moments of inertia for each simple body about the parallel reference axis is the moment of inertia of the composite body.

5. Any cutout area has must be assigned a negative moment; all others are considered positive.

♦ Moment of Inertia of Some Standard Geometrical Shapes

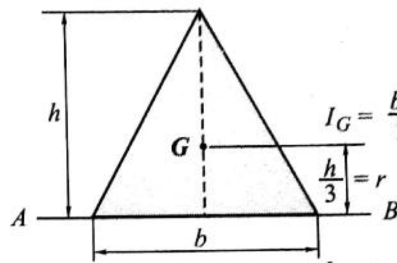
1. Rectangle About its Base



$I_G = \frac{bh^3}{12}$

$I_{AB} = \frac{bh^3}{3}$

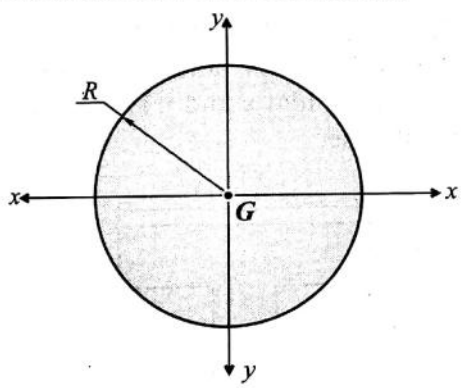
2. Triangle About its Base



$I_G = \frac{bh^3}{36}$

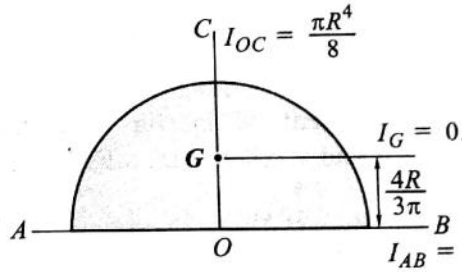
$I_{AB} = \frac{bh^3}{12}$

3. Circle About Diametrical Axis



$$I_G = I_{xx} = I_{yy} = \frac{\pi R^4}{4}$$

4. Semicircle About Diametrical Axis

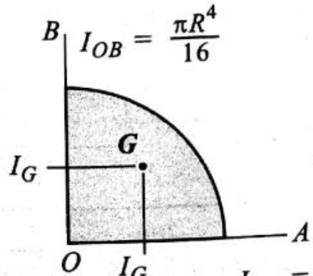


$I_{OC} = \frac{\pi R^4}{8}$

$I_G = 0.11R^4$

$I_{AB} = \frac{\pi R^4}{8}$

5. Quarter Circle



$I_{OB} = \frac{\pi R^4}{16}$

$I_{OA} = \frac{\pi R^4}{16}$

$I_G = 0.055\,R^4$

PRACTICE QUESTIONS

[I] Problems

1. Find the M.I. about the centroidal axis in Fig. 6.E1.

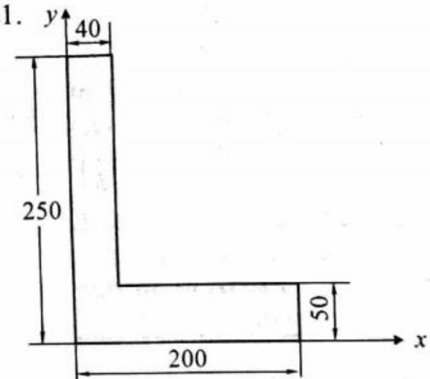
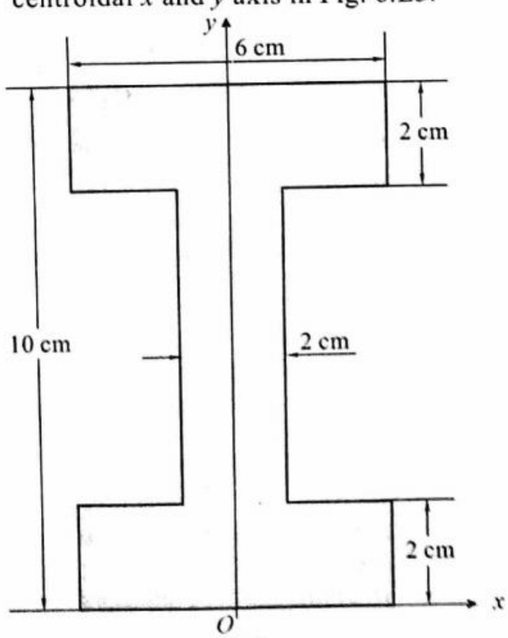


Fig. 6.E1 *[All dimensions are in mm]*

[**Ans.** $\bar{x} = 64.4$ mm, $\bar{y} = 80.5$ mm,
$I_{xx} = 98.18 \times 10^6$ mm^4 and
$I_{yy} = 62.83 \times 10^6$ mm^4.]

3. Find the moment of inertia w.r.t. the centroidal x and y axis in Fig. 6.E3.



[**Ans.** $I_{x_G} = 428$ cm^4 and
$I_{y_G} = 76$ cm^4.]

2. Find the M.I. about the centroidal axis in Fig. 6.E2.

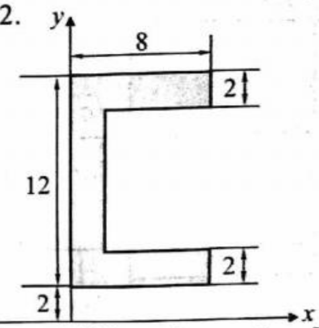
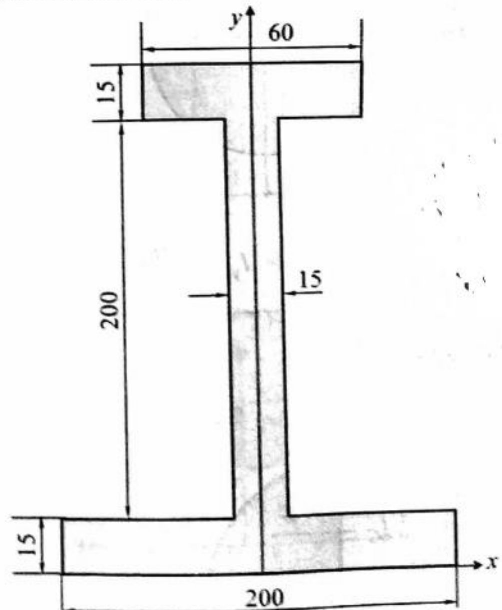


Fig. 6.E2 *[All dimensions are in cm]*

[**Ans.** $I_{xx} = 14.89$ cm^4]

4. Find the centroid of the unequal I- section shown in Fig. 6.E4 and calculate M.I. about the centroidal x and y axis. Also find M.I. about base.



[All dimensions are in mm]

[**Ans.** $\bar{x} = 0$, $\bar{y} = 82.28$ mm,
$I_{x_G} = 47.756 \times 10^6$ mm^4,
$I_{y_G} = 10.326 \times 10^6$ mm^4 and
$I_{Base} = 94.47 \times 10^6$ mm^4.]

REFERENCES

A TEXTBOOK OF ENGINEERING MECHANICS by R.S. KHURMI
MECHANICS OF SOLIDS by S. S. BHAVIKATTI
ENGINEERING MECHANICS by N H Dubey