UNIT II

Statistical Physics
&
Quantum Mechanics

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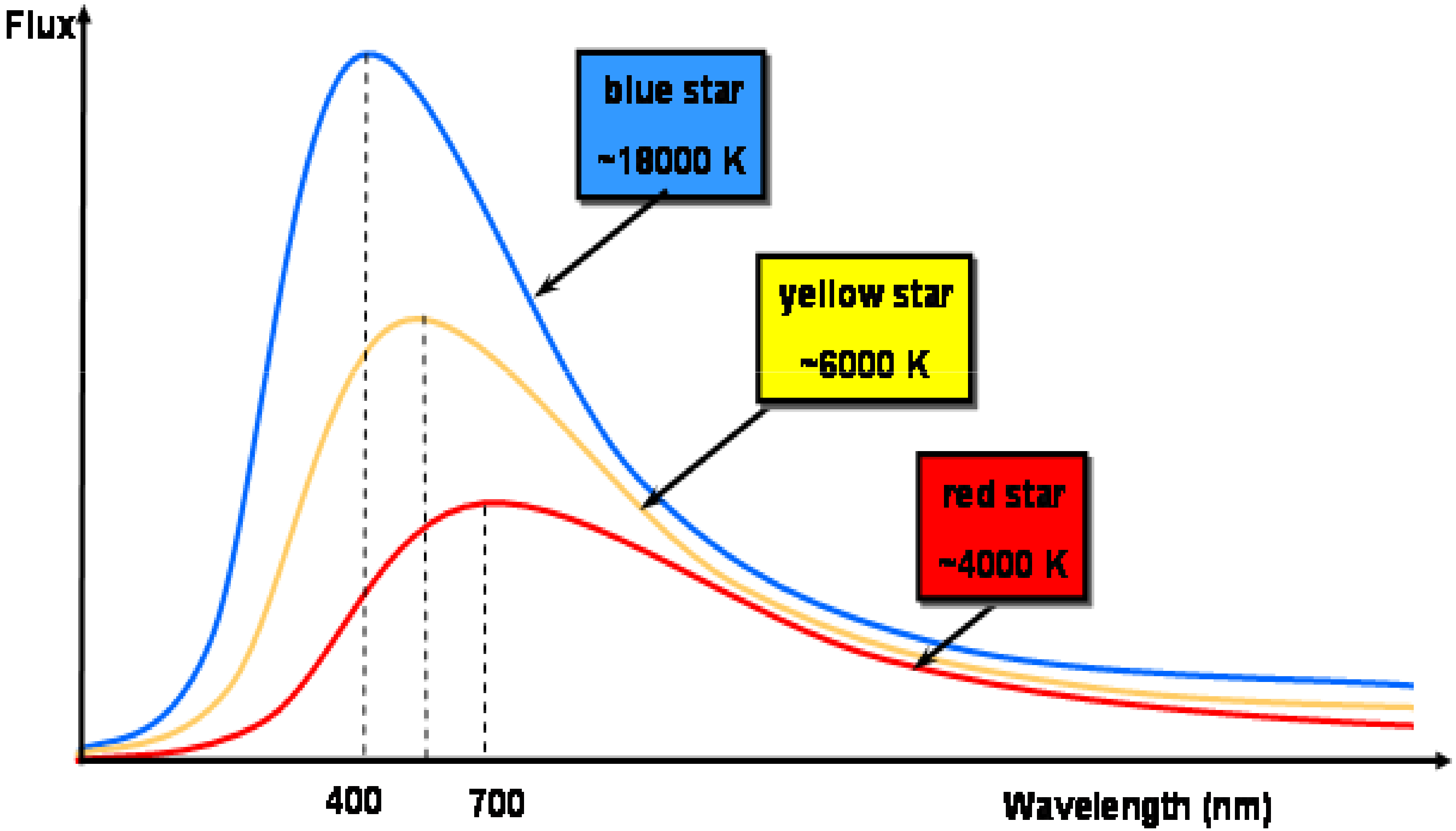
Applied Physics-I

Introduction : Black Body Radiation

- A blackbody is an idealized object which absorbs and emits all frequencies. Classical physics can be used to derive an equation which describes the intensity of blackbody radiation as a function of frequency for a fixed temperature — the result is known as the Rayleigh-Jeans law. Although the Rayleigh-Jeans law works for low frequencies, it diverges as f^2; this divergence for high frequencies is called the ultraviolet catastrophe.
- In 1896 Wien derived a distribution law of radiation. Planck, who was a colleague of Wien's when he was carrying out this work, later, in 1900, based quantum theory on the fact that Wien's law, while valid at high frequencies, broke down completely at low frequencies.
- All objects with a temperature above absolute zero (0 K, -273.15 ^{o}C) emit energy in the form of **electromagnetic radiation**. A **blackbody** is a theoretical or model body which absorbs all radiation falling on it, reflecting or transmitting none. It is a hypothetical object which is a “perfect” absorber and a “perfect” emitter of radiation over all **wavelengths**.

- The spectral distribution of the thermal energy radiated by a blackbody (i.e. the pattern of the intensity of the radiation over a range of wavelengths or frequencies) depends ***only on its temperature***. Blackbody radiation curves at several different temperatures.



- In the image above, notice that:
- The blackbody radiation curves have quite a complex shape (described by Planck's Law).
- The spectral profile (or curve) at a specific temperature corresponds to a specific peak wavelength, and vice versa.
- As the temperature of the blackbody increases, the peak wavelength decreases (Wien's Law).
- The intensity (or **flux**) at all wavelengths increases as the temperature of the blackbody increases.
- The total energy being radiated (the **area** under the curve) increases rapidly as the temperature increases (Stefan–Boltzmann Law).
- Although the intensity may be very low at very short or long wavelengths, at any temperature above absolute zero energy is theoretically emitted at *all* wavelengths (the blackbody radiation curves never reach zero).

- The characteristics of blackbody radiation can be described in terms of several laws:
- 1. **Planck's Law** of blackbody radiation, a formula to determine the spectral energy **density** of the emission at each **wavelength** *(E_λ)* at a particular absolute temperature (T).
- 2. **Wien's Displacement Law**, which states that the **frequency** of the peak of the emission (f_{max}) increases linearly with absolute temperature (T). Conversely, as the temperature of the body *increases*, the wavelength at the emission peak *decreases*.
- 3. **Stefan–Boltzmann Law**, which relates the *total* energy emitted *(E)* to the absolute temperature (T).

- **Planck's Law:** **Assumptions:1.** A cavity in a material that is maintained at constant temperature T. The emission of radiation from the cavity walls is in equilibrium with the radiation that is absorbed by the walls.The radiation field in an empty volume in thermal equilibrium with a container at T can be viewed as a superposition of standing harmonic waves. The radiation field in an empty volume (V=L3) is in thermal equilibrium with container at temperature T.

- 2. Planck assumed that the sources of radiation are atoms in a state of oscillation and that the vibrational energy of each oscillator may have any of a series of discrete values but never any value between. Planck further assumed that when an oscillator changes from a state of energy E_1 to a state of lower energy E_2, the discrete amount of energy $E_1 - E_2$, or quantum of radiation, is equal to the product of the frequency of the radiation, symbolized by the Greek letter ν and a constant h, now called Planck's constant, that he determined from blackbody radiation data;

  i.e., $E_1 - E_2 = h\nu$.

- <u>Derivation of Planck's Radiation Law:</u>

- Let N be the total number of vibrating oscillators & E be their energy, then average energy of these oscillators is given by

  $$\langle E \rangle = E_{tot.}/N_{tot}$$

- if N_0 is the number of oscillators with zero energy (in the so-called ground-state), then the numbers in the 1st, 2nd, 3rd etc. levels () are given by

  $$N_1 = N_0 e^{-E_1/kT},\ N_2 = N_0 e^{-E_2/kT},\ N_3 = N_0 e^{-E_3/kT},\ \cdots$$

- But, as E=nhν, we can write

$$N_1 = N_0 e^{-h\nu/kT},\ N_2 = N_0 e^{-2h\nu/kT},\ N_3 = N_0 e^{-3h\nu/kT}, \cdots$$

To make it easier to write, we are going to substitute, $x = e^{-h\nu/kT}$

so we have, $N_1 = N_0 x,\ N_2 = N_0 x^2,\ N_3 = N_0 x^3, \cdots$

The total number of oscillators is given by

$$N_{tot} = N_0 + N_1 + N_2 + N_3 + \cdots = N_0\left(1 + x + x^2 + x^3 + \cdots\right)$$

Remember, this is the number of oscillators at each frequency, so the energy at each frequency is given by the number at each frequency multiplied by the energy of each oscillator at that frequency. So

$$E_1 = N_1 h\nu,\ E_2 = N_2 2h\nu,\ E_3 = N_3 3h\nu, \cdots$$

which we can now write as

$$E_1 = h\nu N_0 x,\ E_2 = 2h\nu N_0 x^2,\ E_3 = 3h\nu N_0 x^3, \cdots$$

The total energy is given by

$$E_{tot} = E_0 + E_1 + E_2 + E_3 + \cdots = N_0 h\nu(0 + x + 2x^2 + 3x^3 + \cdots)$$

The average energy <E> is given by

$$\langle E \rangle = \frac{E_{tot}}{N_{tot}} = \frac{N_0 h\nu(0+x+2x^2+3x^3+\cdots)}{N_0(1+x+x^2+x^3+\cdots)}$$

where

$$x = e^{-h\nu/kT}$$

Then

$$\boxed{\langle E \rangle = \frac{h\nu}{(1/x - 1)} = \frac{h\nu}{(e^{h\nu/kT} - 1)}}$$

The number of modes in the frequency interval ν to ν +dν is given by (8πν2/c3) dν per unit volume. (By Rayleigh Jeans law.)

- The energy density of radiation in the frequency range ν to ν +dν is

$$u(\nu)\, d\nu = <E> \text{ x No. of modes}$$

$$u(\nu)\, d\nu = \frac{8\pi h\nu 3 \qquad d\nu}{c\ 3\ [\exp(h\nu/kT) - 1]}$$

This is the Planck distribution function.

In terms of wavelength Planck’s radiation formula can be given as

$$E_\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{\exp(hc/kT\lambda) - 1}.$$

Matter Waves

1. According to Plank's quantum theory, energy is emitted in the form of packets or quanta called Photons.

2. According to Plank's law, the energy of photons per unit volume in black body radiation is given by

$$E_\lambda=8\pi hc/\lambda^5[\exp(h\upsilon/kT)-1]$$

- According to Louis de Broglie since radiation such as light exhibits dual nature both wave and particle, the matter must also posses dual nature.
- The wave associated with matter called matter wave has the wavelength $\lambda=h/m\upsilon$ and is called de Broglie wavelength

Characteristics of matter waves

Since $\lambda = h/m\upsilon$,

1. Lighter the particle, greater is the wavelength associated with it.
2. Lesser the velocity of the particle, longer the wavelength associated with it.
3. For $v=0$, $\lambda=\infty$. This means that only with moving particle matter wave is associated.
4. Whether the particle is charged or not, matter wave is associated with it. This reveals that these waves are not electromagnetic but a new kind of waves.
5. No single phenomena exhibits both particle nature and wave nature simultaneously.
6. While position of a particle is confined to a particular location at any time, the matter wave associated with it has some spread as it is a wave. Thus the wave nature of matter introduces an uncertainty in the location of the position of the particle. Heisenberg's uncertainty principle is based on this concept.

deBroglie hypothesis :

The dual nature of light possessing both wave and particle properties was explained by combining Plank's expression for the energy of a photon $E = h\nu$ with Einstein's mass energy relation $E = m c2$ (where c is velocity of light , h is Plank's constant , m is mass of particle)

$$h \nu = m c2$$

Introducing $\nu = c / \lambda$, we get $h c / \lambda = m c2$

==> $\lambda = h / mc = h / p$ where p is momentum of particle

λ is deBroglie wavelength associated with a photon.

deBroglie proposed the concept of matter waves , according to which a material particle of mass 'm' moving with velocity 'v' should be associated with deBroglie wavelength 'λ' given by

$$\lambda = h / m v = h / p$$

The above eqn. represents deBroglie wave eqn.

The Phase Velocity

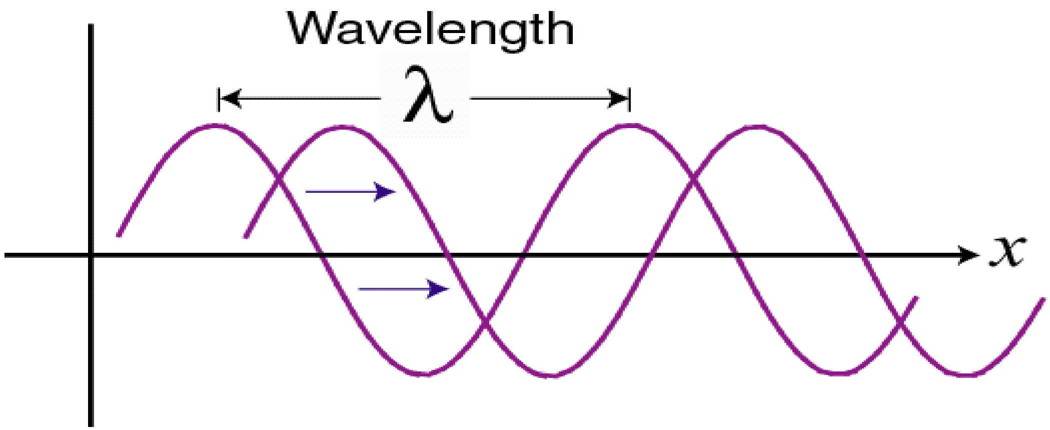
The phase velocity is the wavelength / period: v = λ/ *t*

Since *f* = 1/*t* :   **v = *l f***

In terms of k, *k* = 2p / λ, and

the angular frequency, *w* = 2p / *t*, this is:

v = *w* / *k*



Davisson-Germer Experiment

- In order to test de Broglie's hypothesis that matter behaved like waves, Davisson and Germer set up an experiment very similar to what might be used to look at the interference pattern from x-rays scattering from a crystal surface. The basic idea is that the planar nature of crystal structure provides scattering surfaces at regular intervals, thus waves that scatter from one surface can constructively or destructively interfere from waves that scatter from the next crystal plane deeper into the crystal. Their experimental apparatus is shown below

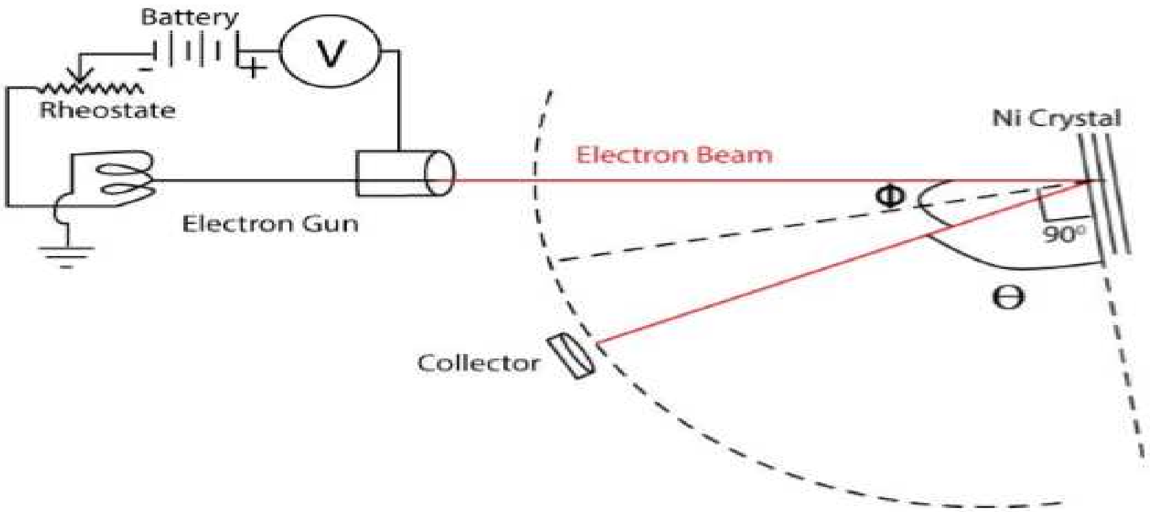


Figure shows experimental arrangement for electron diffraction.

- Apparatus
- Electron gun.
- Nickel crystal.
- Electron detector/collector.
- **Working**
- Electron gun has tungsten filament coated with barium oxide for high emission efficiency. When this filament is heated it emits thermal electrons.

  The electrons are accelerated by cylindrical shield kept at fixed known high positive voltage (V). The electrons emerge out of shield as fine beam and its energy can be calculated using value of voltage applied. This beam is made to fall on the surface of nickel crystal. The electron beam gets reflected after hitting the nickel crystal. The intensity of reflected electrons in a particular direction is measured by the electron collector, which can be moved on a circular scale. The collector provides the value of current which is proportionate to the number of electrons incident on it.The intensity of reflected electron beam is recorded for different angles of deflection (Φ) and different velocities of electrons which is measured by applied voltage (V).A radial graph is plotted to observe the results of recorded data. It is observed that, current is maximum when deflection angle $\Phi = 50°$ and accelerating voltage $V = 54V$.

- According to Bragg's diffraction formula −
- **Path difference = wavelenght i.e.   asinθ = λ**
- Where, a = opening for wave to enter. λ = Wavelength

But in Davisson and Germer's experiment,

Path difference = 2dsinθ

∴ 2dsinθ = λ

Where, d = Spacing between atomic planes. λ = wavelenght

θ = Glancing angle. For nickel crystal spacing between atomic planes is d = 0.91×10^{-10}, For deflection angle $\Phi = 50^o$, Glancing angle is $\theta = 65^o$ as −**glancing angle = $90^o - \Phi/2$**

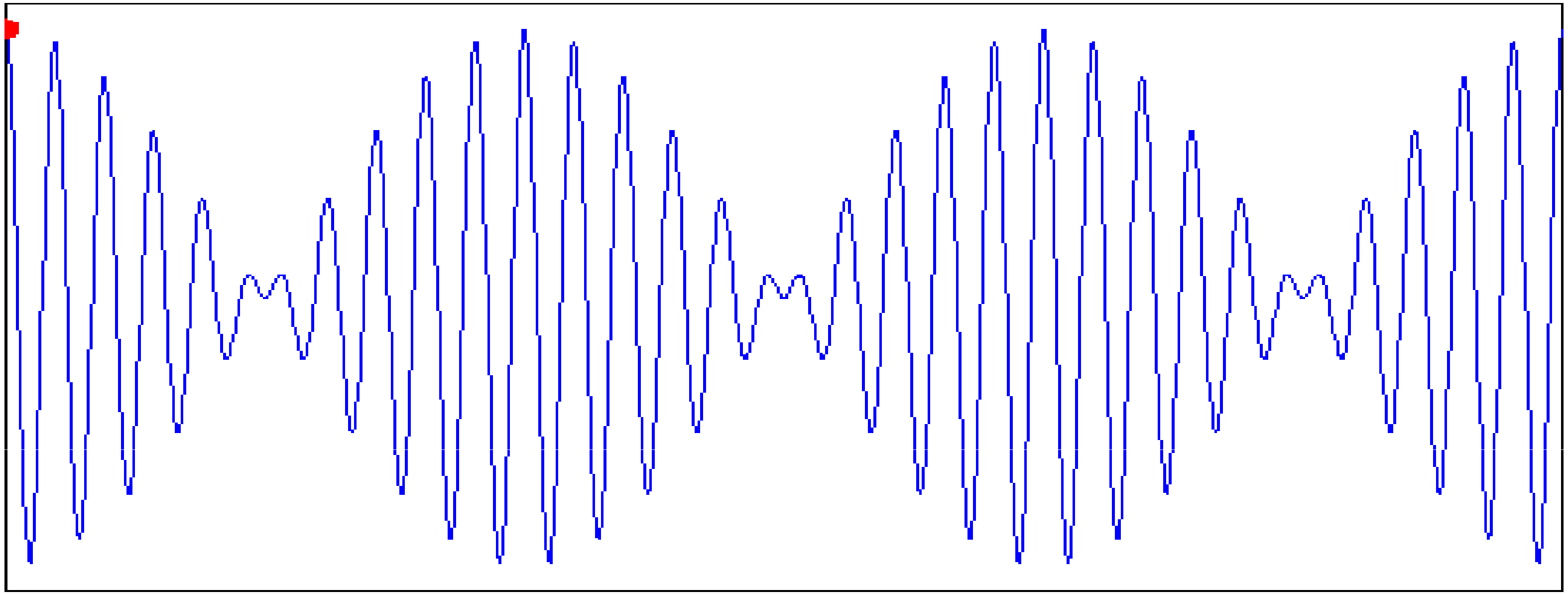
Therefore,

$\lambda = 2 \times 0.91 \times 10^{-10} \times \sin(65) = \lambda = 1.65A$

By using de Broglie's wavelength of electron at accelerating voltage 54V is given by − **$\lambda = 12.27\sqrt{v}A = 12.27\sqrt{54} = 1.66A$**

This confirms the wave nature electron and proves that materials have dual nature.

Group Velocity



This is the velocity at which the overall shape of the wave's amplitudes, or the wave 'envelope', propagates. (= ***signal velocity*****)**

Here, phase velocity = group velocity (the medium is ***non-dispersive)***

Phase Velocity

The distinction between the phase velocity and the group velocity of a wave is a concept of general signicance for many different waves in physics: electromagnetic waves, particle waves, elastic waves and so on. We start by considering a general one-dimensional wave

$A(x; t) = A0e^{i(kx-\omega t)}$

where A0 is the amplitude, k the wave number, ω is the angular frequency, and t the time. The important thing to note is that ω depends on the wave number k or the wavelength $\lambda = 2\Pi/k$. This phenomenon is called dispersion and it might be familiar from optics, where the speed of light in a material (or the index of refraction) depends on the wavelength.

Starting from the dispersion $\omega(k)$, we derive the phase velocity as

$$vp = \omega /k$$

and the group velocity as

$$vg = d\omega /dk$$

Albert Einstein first explained the wave–particle duality of light in 1905. Louis de Broglie hypothesized that any particle should also exhibit such a duality. The velocity of a particle, he concluded then (but may be questioned today, see above), should always equal the group velocity of the corresponding wave. De Broglie deduced that if the duality equations already known for light were the same for any particle, then his hypothesis would hold. This means that

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\partial (E/\hbar)}{\partial (p/\hbar)} = \frac{\partial E}{\partial p}$$

where E is the total energy of the particle, p is its momentum, $\hbar$ is the reduced Planck **constant**. For a free non-relativistic particle it follows that

$$\begin{aligned} v_g &= \frac{\partial E}{\partial p} = \frac{\partial}{\partial p}\left(\frac{1}{2}\frac{p^2}{m}\right), \\ &= \frac{p}{m}, \\ &= v. \end{aligned}$$

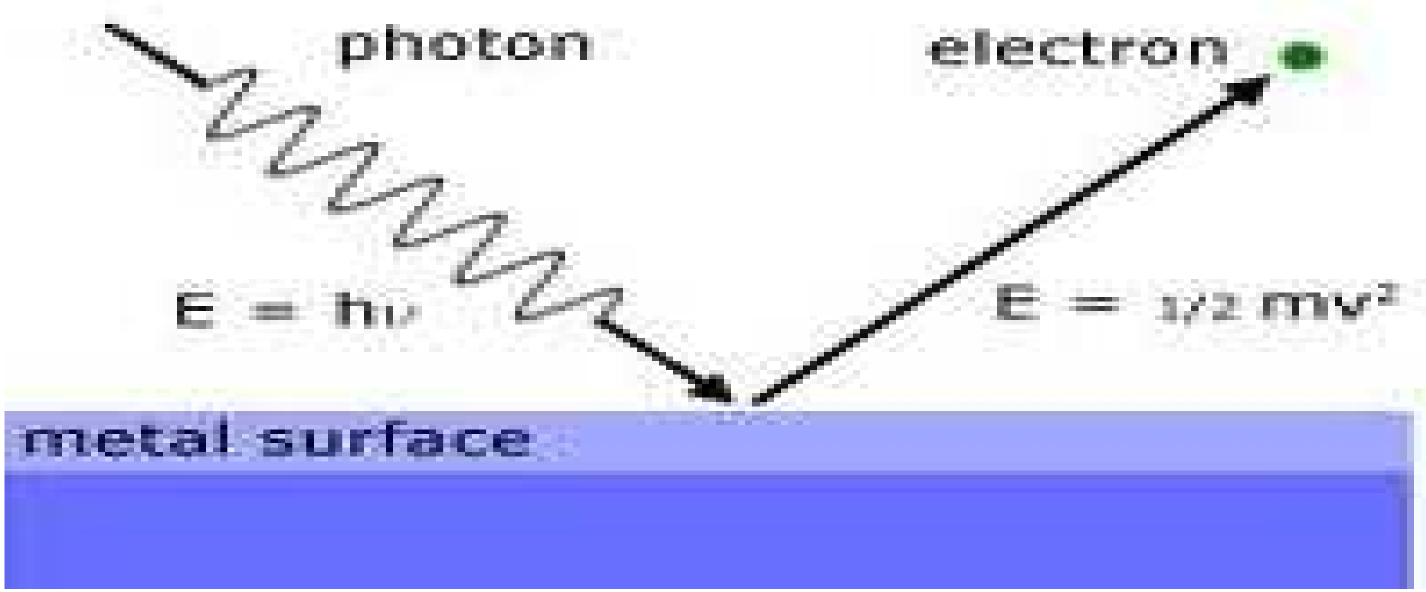
where   is the *mass of the particle and v* its velocity.

$$
\begin{aligned}
v_g &= \frac{\partial E}{\partial p} = \frac{\partial}{\partial p}\left(\sqrt{p^2c^2+m^2c^4}\right), \\
&= \frac{pc^2}{\sqrt{p^2c^2+m^2c^4}}, \\
&= \frac{p}{m\sqrt{\left(\frac{p}{mc}\right)^2+1}}, \\
&= \frac{p}{m\gamma}, \\
&= \frac{mv\gamma}{m\gamma}, \\
&= v.
\end{aligned}
$$

where *m* is the mass of the particle, *c* is the speed of light in a vacuum, γ is the Lorentz factor, and *v* is the velocity of the particle regardless of wave behavior.
Group velocity (equal to an electron's speed) should not be confused with phase velocity (equal to the product of the electron's frequency multiplied by its wavelength).
Both in relativistic and non-relativistic quantum physics, we can identify the group velocity of a particle's wave function with the particle velocity. Quantum mechanics has very accurately demonstrated this hypothesis, and the relation has been shown explicitly for particles as large as molecules

PHOTOELECTRIC EFFECT

In photoelectric effect, electrons are emitted from matter (metals and non-metallic solids, liquids or gases) as a consequence of their absorption of energy from electromagnetic radiation of very short wavelength and high frequency, such as ultraviolet radiation.



MATHEMATICAL DESCRIPTION

The maximum kinetic energy of an ejected electron is given by:

$$K_{max} = hf - \Phi$$

where h is the Planck constant and f is the frequency of the incident photon. The term Φ = is the work function which gives the minimum energy required to remove a delocalized electron from the surface of the metal. The work function satisfies $\Phi = hf_0$

where f_0 is the threshold frequency for the metal. The maximum kinetic energy of an ejected electron is then

$$K_{max} = hf - hf_0$$

Kinetic energy is positive, so we must have $f > f_0$ for the photoelectric effect to occur.

COMPTON EFFECT

Compton scattering is an inelastic scattering of a photon by a free charged particle, usually an electron. It results in a decrease in energy (increase in wavelength) of the photon (which may be an X-ray or gamma ray photon), called the **Compton effect**. Part of the energy of the photon is transferred to the scattering electron. **Inverse Compton scattering** also exists, in which a charged particle transfers part of its energy to a photon.

MATHEMATICAL DESCRIPTION

ENERGY CONSERVATION

Energy of photon before collision = $E_1 = hu_1$
Energy of electron before collision = $E_2 = m_o c^2$
Energy of photon after collision = $E1` = hu_2$
Energy of electron after collision = $E2` = mc^2$

Since the collision between photon and electron is elastic. Therefore energy and momentum will be conserved.

Total energy before collision = Total energy after collision

$$E_1 + E_2 = E_1` + E_2`$$

$$hu_1 + m_o c^2 = hu_2 + mc^2$$

$$hu_1 - hu_2 = mc^2 - m_o c^2$$

$$h(u_1 - u_2) = c^2(m - m_o)\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

MOMENTUM CONSERVATION

Momentum of photon before collision $= h/l_1$
Momentum of electron before collision $= 0$
Momentum of photon after collision $= h/l_2$
Momentum of electron after collision $= mv$

MOMENTUM EQUATION ALONG X-AXIS

$$h/l_1 + 0 = h/l_2 Cosq + mvCosf$$

$$h/l_1 = h/l_2 Cosq + mvCosf \ldots\ldots\ldots\ldots(2)$$

MOMENTUM EQUATION ALONG Y-AXIS

$$0 + 0 = h/l_2 Sinq + (-mvSinf)$$

$$h/l_2 Sinq - mvSinf = 0 \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Solving (1) , (2) and (3), we get the following result:

$$1/u_2 - 1/u_1 = h/m_oc^2 \text{ (1- Cosq) } \ldots\ldots\ldots\ldots.(4)$$

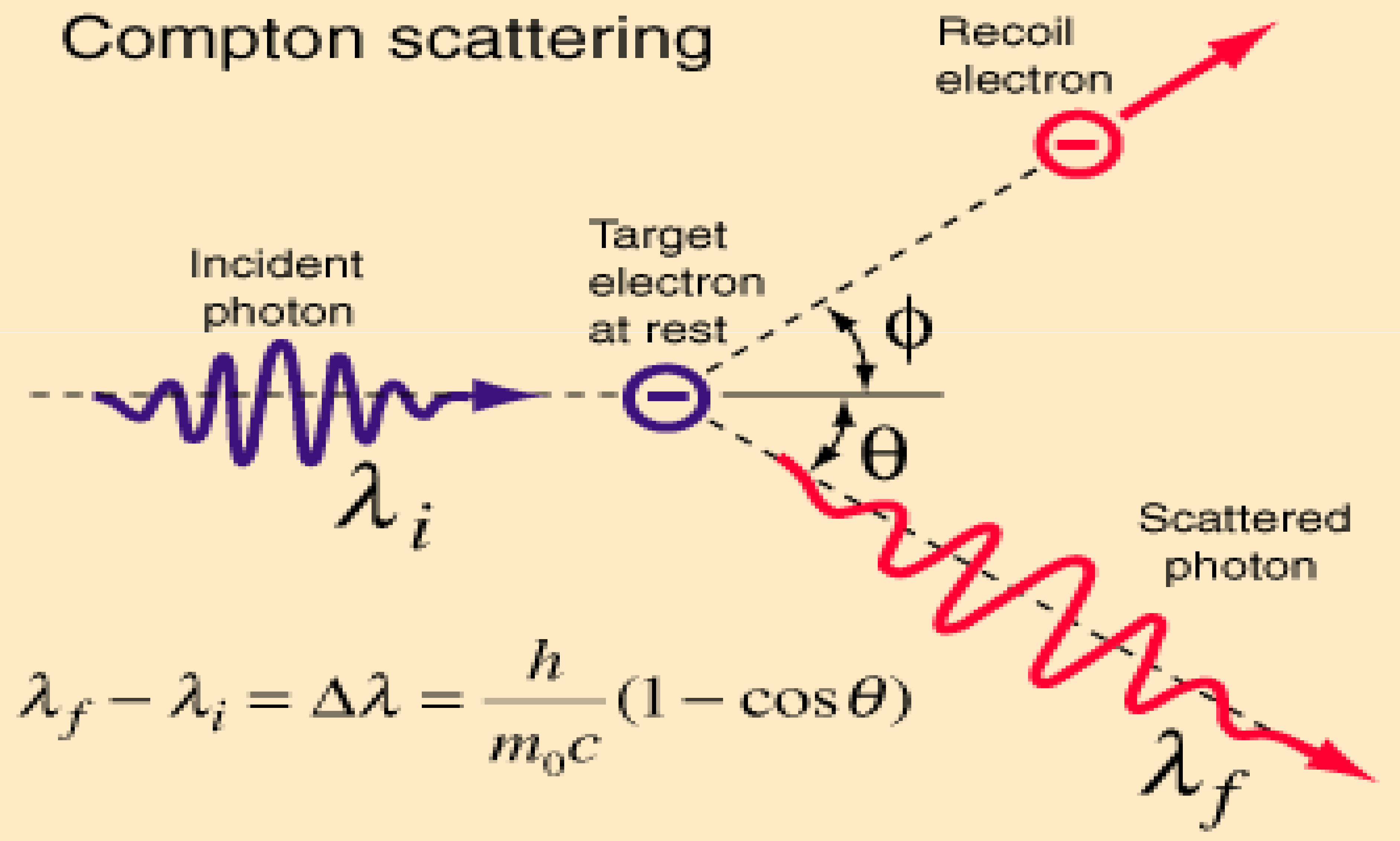
$$c(1/u_2 - 1/u_1) = hc/m_oc^2 \text{ (1- Cosq)}$$

$$c/u_2 - c/u_1 = h/m_oc \text{ (1- Cosq)}$$

But $c/u = l$, therefore,

$$l_2 - l_1 = h/m_oc \text{ (1- Cosq)}$$

where $l_2 - l_1$ = Compton's shift in wavelength
h/m_oc is called Compton's wavelength and its value is
$2.426X10^{-12\text{ m}}$

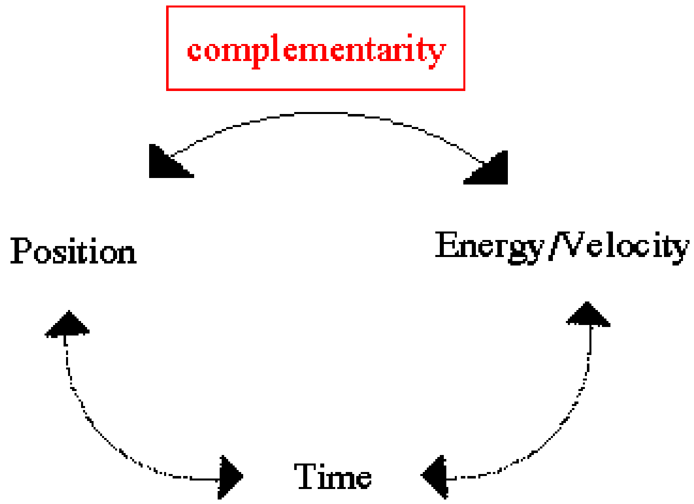

Compton scattering
Recoil electron
Target electron at rest
Incident photon
ϕ
θ
λ_i
Scattered photon
λ_f
$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$

Heisenberg's uncertainty principle

•The uncertainty principle states that the position and velocity cannot both be measured,exactly, at the same time (actually pairs of position, energy and time)

•uncertainty principle derives from the measurement problem, the intimate connection between the wave and particle nature of quantum objects.

•the change in a velocity of a particle becomes more ill defined as the wave function is confined to a smaller region.



Mathematically we describe the uncertainty principle as the following, where `x' is position and `p' is momentum:

$$\Delta x \Delta p > \frac{h}{2\pi}$$

This is perhaps the most famous equation next to $E=mc^2$ in physics. It basically says that the combination of the error in position times the error in momentum must always be greater than Planck's constant. So, you can measure the position of an electron to some accuracy, but then its momentum will be inside a very large range of values. Likewise, you can measure the momentum precisely, but then its position is unknown.Notice that this is not the measurement problem in another form, the combination of position, energy (momentum) and time are actually undefined for a quantum particle until a measurement is made (then the wave function collapses).

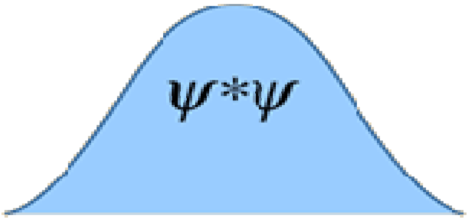
Also notice that the uncertainty principle is unimportant to macroscopic objects since Planck's constant, h, is so small (10^{-34}). For example, the uncertainty in position of a thrown baseball is 10^{-30} millimeters.

The depth of the uncertainty principle is realized when we ask the question; is our knowledge of reality unlimited? The answer is no, because the uncertainty principle states that there is a built-in uncertainty, indeterminacy, unpredictability to Nature.

Heisenberg Example: Electron can not Confined in the Nucleus

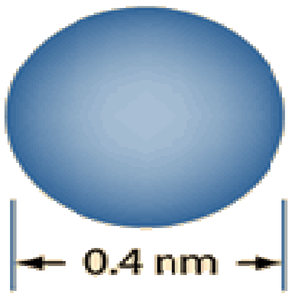
The uncertainty principle contains implications about the energy that would be required to contain a particle within a given volume. The energy required to contain particles comes from the fundamental forces, and in particular the electromagnetic force provides the attraction necessary to contain electrons within the atom, and the strong nuclear force provides the attraction necessary to contain particles within the nucleus. But Planck's constant, appearing in the uncertainty principle, determines the size of the confinement that can be produced by these forces. Another way of saying it is that the strengths of the nuclear and electromagnetic forces along with the constraint embodied in the value of Planck's constant determine the scales of the atom and the nucleus.

The following very approximate calculation serves to give an order of magnitude for the energies required to contain particles.



$\psi^*\psi$ is the probability of finding the particle.

ψ = wavefunction



Assume $\Delta p = p$

$$E = \frac{p^2}{2m}$$

Assume atomic size = $0.4nm$

Nuclear size = $\frac{1}{20,000} x \; 0.4nm$

Using the atomic size as the uncertainty in position:

This shows that Planck's constant determines the relationship between Δx and Δp and therefore the energy of confinement.

$$\Delta p = \frac{h}{\Delta x} = 1.66x10^{-24} kg \cdot m / s$$

These are in the range of observed atomic and nuclear processes.

Energy to:

Confine electron in atom: $9.4eV$

Confine proton in nucleus: $2.05MeV$

Confine electron in nucleus: $3.77GeV$

This is about a factor of a thousand above the observed energies of nuclear processes, indicating that the electron cannot be confined in the nucleus!

The Time-Independent Schrödinger Equation

We start with the one-dimensional classical wave equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

By introducing the separation of variables

$$u(x,t) - \psi(x)f(t)$$

we obtain

$$f(t)\frac{d^2\psi(x)}{dx^2} = \frac{1}{v^2}\psi(x)\frac{d^2 f(t)}{dt^2}$$

If we introduce one of the standard wave equation solutions for f(t)

$$\frac{d^2\psi(x)}{dx^2} = \frac{-\omega^2}{v^2}\psi(x)$$

Now we have an ordinary differential equation describing the spatial amplitude of the matter wave as a function of position. The energy of a particle is the sum of kinetic and potential parts

$$E = \frac{p^2}{2m} + V(x)$$

which can be solved for the momentum, to obtain

$$p = \{2m[E - V(x)]\}^{1/2}$$

Now we can use the de Broglie formula to get an expression for the wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\{2m[E - V(x)]\}^{1/2}}$$

if $\omega = 2\pi\nu$ then

$$\frac{\omega^2}{v^2} = \frac{4\pi^2\nu^2}{v^2} = \frac{4\pi^2}{\lambda^2} = \frac{2m[E - V(x)]}{\hbar^2}$$

we obtain the famous *time-independent Schrödinger equation*

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0$$

Schrodinger Time Dependent: A Wave Equation for Electrons

$$E\psi = \hbar\omega\psi = -j\hbar\frac{\partial}{\partial t}\psi$$

$$E = \frac{p^2}{2m}$$ (free-particle)

$$-j\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t)$$

This is time-Independent Schrodinger Wave Equation

Physical significance of Wave function Ψ

1. The wave functions Ψ_n and the corresponding energies E_n, which are often called eigen functions and eigen values respectively, describe the quantum state of the particle.

2. The wave function Ψ has no direct physical meaning. It is a complex quantity representing the variation of matter wave. It connects the particle nature and its associated wave nature.

3. $\Psi\Psi^*$ or $|\Psi|^2$ is the probability density function. $\Psi\Psi^* dxdydz$ gives the probability of finding the electron in the region of space between x and x+dx, y and y+dy and z and z+dz.If the particle is present

$$\int \Psi\Psi^* dxdydz = 1$$

4. It can be considered as probability amplitude since it is used to find the location of the particle.

- Schrodinger developed a differential equation whose solutions yield the possible wave functions that can be associated with a particle in a given situation.

Schrödinger wave equation

The equation tells us how the wave function changes as a result of forces acting on the particle.

The one dimensional time independent Schrödinger wave equation is given by

$$d^2\Psi/dx^2 + [2m(E-V)/\hbar^2]\,\Psi=0$$

(or)

$$d^2\Psi/dx^2 + [8\pi^2 m(E-V)/h^2]\,\Psi=0$$

This equation is popularly known as schrodinger equation.

Particle in one dimensional potential box

Quantum mechanics has many applications in atomic physics.

Consider one dimensional potential well of width L.

Let the potential V=0inside the well and V= ∞outside the well.

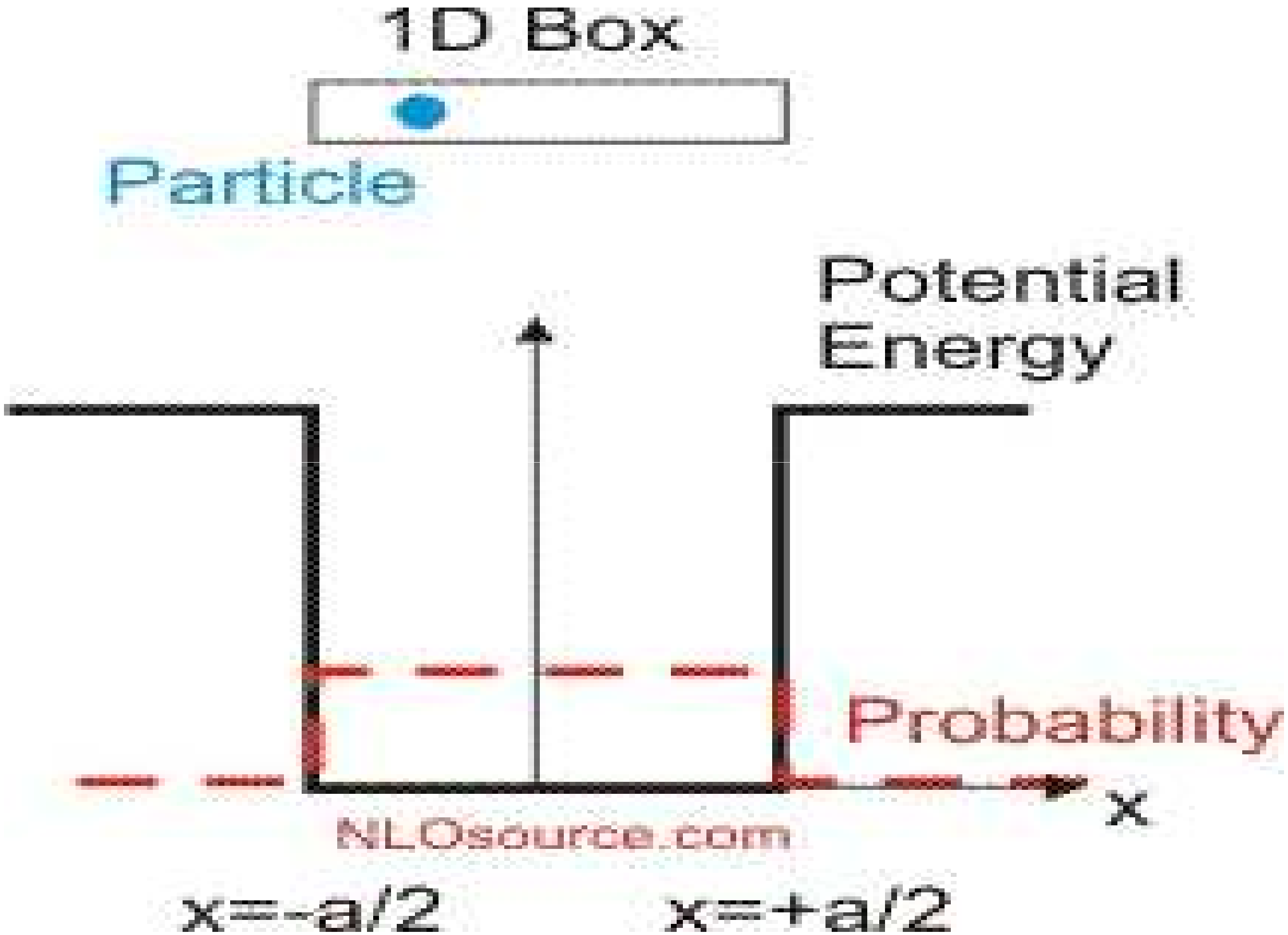
Substituting these values in Schrödinger wave equation and simplifying we get the energy of the n^{th} quantum level,

$E_n=(n^2\pi^2\hbar^2)/2mL^2=n^2h^2/8mL^2$

When the particle is in a potential well of width L, $\Psi_n=(\sqrt{2/L})\sin(n\pi/L)x$ & $E_n = n^2h^2/8mL^2$, n=1,2,3,….

When the particle is in a potential box of sides L_x, L_y, L_z $\Psi_n=(\sqrt{8/V})\sin(n_x \pi/L_x) x \sin(n_y \pi/L_y) y \sin(n_z \pi/L_z) z$.

Where n_x, n_y or n_z is an integer under the constraint $n2= n_x^2+n_y^2+ n_z^2$.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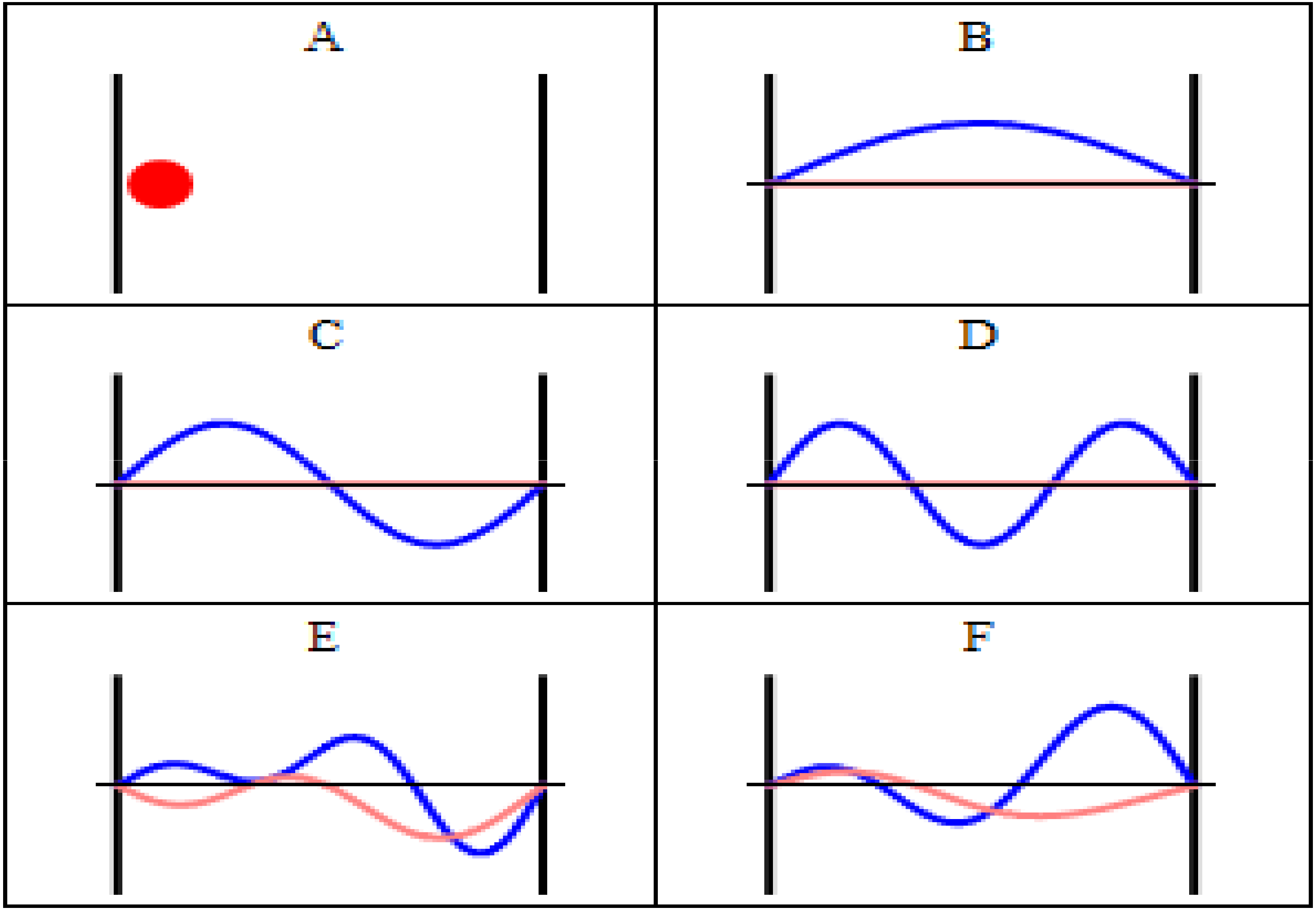
Particle
Potential
Energy
Probability
x
NLOsource.com
x=-a/2
x=+a/2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C
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E
F

Classical statistics

In classical mechanics all the particles (fundamental and composite particles, atoms, molecules, electrons, etc.) in the system are considered distinguishable. This means that one can label and track each individual particle in a system. As a consequence changing the position of any two particles in the system leads to a completely different configuration of the entire system. Furthermore there is no restriction on placing more than one particle in any given state accessible to the system. Classical statistics is called Maxwell–Boltzmann statistics (or M–B statistics).

Quantum statistics

The fundamental feature of quantum mechanics that distinguishes it from classical mechanics is that particles of a particular type are indistinguishable from one another. This means that in an assembly consisting of similar particles, interchanging any two particles does not lead to a new configuration of the system (in the language of quantum mechanics: the wave function of the system is invariant with respect to the interchange of the constituent particles). In case of a system consisting of particles belonging to different nature (for example electrons and protons), the wave function of the system is invariant separately for the assembly of the two particles.

Bose–Einstein statistics

While this difference between classical and quantum description of systems is fundamental to all of quantum statistics, it is further divided into the following two classes on the basis of symmetry of the system.

Bose–Einstein statistics

In Bose–Einstein statistics (B–E Statistics) interchanging any two particles of the system leaves the resultant system in a symmetric state. That is, **the wave function of the system before interchanging equals the wave function of the system after interchanging.**

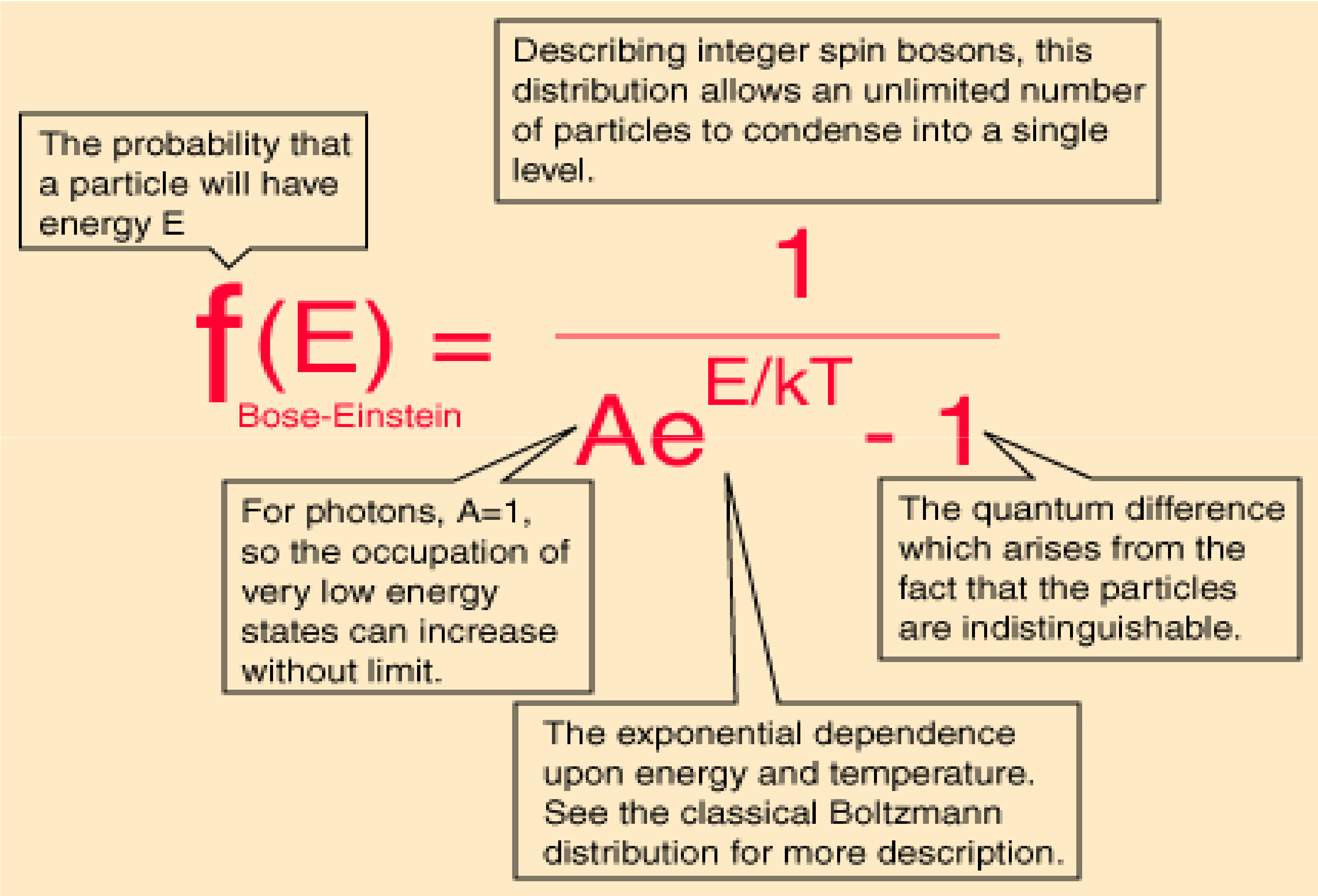
Fermi–Dirac statistics

In Fermi–Dirac statistics (F–D statistics) interchanging any two particles of the system leaves the resultant system in an anti symmetric state. That is, the wave function of the system before interchanging is the wave function of the system after interchanging, with an overall minus sign.

Bose-Einstein Distribution

The Bose-Einstein distribution describes the statistical behavior of integer spin particles (bosons). At low temperatures, bosons can behave very differently than fermions because an unlimited number of them can collect into the same energy state, a phenomenon called "condensation".

$$f(E) = \frac{1}{Ae^{E/kT} - 1}$$



The probability that a particle will have energy E
Describing integer spin bosons, this distribution allows an unlimited number of particles to condense into a single level.
$$f(E)_{\text{Bose-Einstein}} = \frac{1}{Ae^{E/kT} - 1}$$
For photons, A=1, so the occupation of very low energy states can increase without limit.
The exponential dependence upon energy and temperature. See the classical Boltzmann distribution for more description.
The quantum difference which arises from the fact that the particles are indistinguishable.

Fermi-Dirac Distribution

The Fermi-Dirac distribution applies to fermions, particles with half-integer spin which must obey the Pauli exclusion principle. Each type of distribution function has a normalization term multiplying the exponential in the denominator which may be temperature dependent. For the Fermi-Dirac case, that term is usually written:

$$e^{-E_F/kT} \quad \text{where } E_F = \text{Fermi energy}$$

The significance of the Fermi energy is most clearly seem by setting T=0. At absolute zero, the probability is =1 for energies less than the Fermi energy and zero for energies greater than the Fermi energy. We picture all the levels up to the Fermi energy as filled, but no particle has a greater energy. This is entirely consistent with the Pauli exclusion principle where each quantum state can have one but only one particle.

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

At absolute zero, fermions will fill up all available energy states below a level E_F called the Fermi energy with one (and only one) particle. They are constrained by the Pauli exclusion principle. At higher temperatures, some are elevated to levels above the Fermi level.

The probability that a particle will have energy E

$$f(E)_{\text{Fermi-Dirac}} = \frac{1}{e^{(E - E_F)/kT} + 1}$$

See the Maxwell-Boltzmann distribution for a general discussion of the exponential term.

For low temperatures, those energy states below the Fermi energy E_F have a probability of essentially 1, and those above the Fermi energy essentially zero.

The quantum difference which arises from the fact that the particles are indistinguishable.

Assignments

Q. No.1. Why don’t hot objects emit more ultraviolet light than they do?

Q. No.2. What is Photon ?

Q.No. 3. How come *I’ve* never seen a photon?

Q. No.4. What is quantum wavelength of an object ?

Q.No.5. How the mathematical machinery of quantum mechanics is turned to the hydrogen atom, the solutions yield energy levels in exact agreement with the optical spectrum. Explain.

Q.No. 6. Explain uncertainity in energy & time.

Q.No. 7. What is tunling effect ?

Q.No. 8. Define Bosons & fermions.

Q.No. 9. Establish relation between phase velocity & group velocity in dispersive medium.

Q.No. 10. Write S.W. equation for a particle in One-D box.

Tutorial Sheet

1. Black body radiation and Planck's law: Consider a black body maintained at the temperature T . According to Planck's radiation law, the energy per unit volume within the frequency range ν and ν + dν associated with the electromagnetic radiation emitted by the black body is given by $u_\nu \, d\nu = 8 \pi h\nu^3 \, d\nu / c^3 \exp(h \nu / k_B T) - 1$

where h and k_B denote the Planck and the Boltzmann constants, respectively, while c represents the speed of light.

Arrive at the Wien's law, viz. that λ_{MAX} T = b = constant, from the above Planck's radiation law. Note that λ_{MAX} denotes the wavelength at which the energy density of radiation from the black body is the maximum.

Find the total energy emitted by the black body is described by the integral 0 to ∞, Using the above expression for u_ν , evaluate the integral and show that u = $^4{}_c{}^{\sigma/}$ T^4.

2. Consider the emission of electrons due to photoelectric effect from a zinc plate. The work function of zinc is known to be 3.6 eV. What is the maximum energy of the electrons ejected when ultra-violet light of wavelength 3000 A is incident on the zinc plate?

3. Consider a quantum mechanical particle propagating in a given potential and described by the wave function $\Psi(x, t)$. The probability $P(x, t)$ of finding the particle at the position x and the time t is given by

$P(x, t) = |\Psi(x, t)|^2$.

Using the one-dimensional Schrodinger equation, show that the probability $P(x, t)$ satisfies the conservation law

∂P (x, t) + ∂j(x, t) = 0/∂t ∂x

where the quantity j(x, t) represents the conserved current given by

j(x, t) = 2 i m

4 Consider a particle in the infinite square well. Let the initial wave function of the particle be given by

$$\Psi(x, 0) = A\,[\psi_1(x) + \psi_2(x)]\,,$$

where $\psi_1(x)$ and $\psi_2(x)$ denote the ground and the first excited states of the particle.

a) Normalize the wave function $\Psi(x, 0)$.

b) Obtain the wave function at a later time t, viz. $\Psi(x, t)$, and show that the probability $|\Psi(x, t)|^2$ is an oscillating function of time.

c) Evaluate the expectation value of the position in the state $\Psi(x, t)$ and show that it oscillates. What are the angular frequency and the amplitude of the oscillation?

d) What will be the values that you will obtain if you measure the energy of the particle? What are the probabilities for obtaining these values?

e) Evaluate the expectation value of the Hamiltonian operator corresponding to the particle in the state $\Psi(x, t)$. How does it compare with the energy eigen values of the ground and the first excited states?